10B 2SamT-test

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Textbook: Devore 8e

B Two-sample T-test

[ToC]

B.1 When S_1, S_2 must be used instead of σ_1, σ_2 and $n < 40$

One-sample case,

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1) \qquad \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$$

That's why z-test became t-test.

Two-sample case,

$$\frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1) \qquad \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t(\nu)$$

Only difference of the degrees of freedom ν.

B.2 Formula for the Degrees of freedom

The degrees of freedom ν can be calculated by the formula

$$\nu = \frac{(a+b)^2}{\frac{a^2}{n_1-1} + \frac{b^2}{n_2-1}} \qquad \text{where} \qquad a = \frac{S_1^2}{n_1}, \qquad b = \frac{S_2^2}{n_2}.$$

If you are in hurry, you can use $\min(n_1, n_2) - 1$.

B.3 Example:

Two new methods for producing a tire have been proposed. To ascertain which is superior, a tire manufacturer produces a sample of 10 tires using the first method and a sample of 8 using the second. The first set is to be road tested at location A and the second at location B.

It is known that the lifetimes of tires tested at location A or B are normally distributed.

Manufacturer is interested in testing the hypothesis that there is no appreciable difference in the mean life of tires produced by either method.

B.4 Data:

```
Tire Lives in Units of 100 Kilometers

Tires_at_A      Tires_at_B
61.1            62.2
58.2            56.6
62.3            66.4
64              56.2
59.7            57.4
66.2            58.4
57.8            57.6
61.4            65.4
62.2
63.6

X <- c(61.1, 58.2, 62.3, 64, 59.7, 66.2, 57.8, 61.4, 62.2, 63.6);
Y <- c(62.2, 56.6, 66.4, 56.2, 57.4, 58.4, 57.6, 65.4);

t.test(X,Y)
```

$\bar{X} = 61.65 \qquad \bar{Y} = 60.03$ $\qquad \leftarrow$ Is this an meaningful difference?

$S_1^2 = 2.62 \qquad S_2^2 = 4.07.$

We want to test

$$H_0: \quad \mu_1 - \mu_2 = 0$$
$$H_A: \quad \mu_1 - \mu_2 > 0 \quad (upper - tailed)$$

Test Statistic

$$z = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{61.65 - 60.03 - 0}{\sqrt{\frac{1}{10} + \frac{1}{8}}} = .979$$

P-value $= 1 - \Phi(.6124) = .27$.
Can't reject H_0 for $\alpha = .05$. $\Rightarrow$ Not enough evidence to claim Radial tire has higher milage on average.
(Difference is not significant)

B.5 Ex: Arizona Water

(Montogomery p.342) Arsenic concentration in public drinking water supplies is a potential health risk. An article in the Arizona Republic (Sunday, May 27, 2001) reported drinking water arsenic concentrations in parts per billion (ppb) for 10 methropolitan Phoenix communities and 10 communities in rural Arizona.

```
Metro  Phoenix          Rural  Arizona
Phoenix         3       Rimrock        48
Chandler        7       Goodyear       44
Gilbert        25       New_River      40
Glendale       10       Apachie_Jtn    38
Mesa           15       Buckeye        33
Paradise_Vly    6       Nogales        21
Peoria         12       Black_Canyon   20
Scottsdale     25       Sedona         12
Tempe          15       Payson          1
Sun_City        7       Casa_Grande    18

X=c(3,7,25,10,15,6,12,25,15,7)
Y=c(48,44,40,38,33,21,20,12,1,18)
t.test(X, Y)

mean of x  12.5      sd(X)   7.6
mean of y  27.5      sd(Y)  15.3
```

B.6 Ex: Battery Life

- Duracell Alkaline AA batteries vs Eveready Energizer Alkaline AA batteries. 4.5 hours and 4.2 hours, respectively.
- both sample size are 150
- The population standard deviations of lifetime are 1.8 hours for Duracell and 2.0 hours for Eveready batteries.

Test, with significance level of .05, the hypothesis of true mean lifetime of Duracell batteries is longer than that of Eveready brand.

```
x <- 4.5
y <- 4.2
m <- 150
n <- 150
s1 <- 1.8
s2 <- 2
(x-y - 0) / sqrt( s1^2/m + s2^2/n )

test stat  1.365519

1 - pnorm( 1.96 - (.5 - 0) / sqrt( s1^2/m + s2^2/n ))
```

B.7 Example:

- Stopping distances from 50 mph for two different types of braking systems.
- System A: $n_1 = 6$, $\overline{X} = 76$, $S_1 = 5$,
- System B: $n_2 = 6$, $\overline{Y} = 88$, $S_2 = 5.5$.

Use the two-sample $t-$ test at significance level .01 to see if the data showes evidence that the population mean stopping distance of the breaking system A is more than 10 meter shorter than that of system B.

```
x <- 76
y <- 95
m <- 16
n <- 20
s1 <- 12.3
s2 <- 11.5

(x-y - (-10)) / sqrt( s1^2/m + s2^2/n )

pt( -2.01 - (-20 - (-10) ) / sqrt( s1^2/m + s2^2/n ), 15)
```