2D Prob through Set Op

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2D Probability through Set Operations

[ToC]

D.1 Set Operations

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\},$$

 $A = \{1, 3, 5, 7\}, \qquad B = \{2, 4, 5\} \qquad D = \{6, 8\}$

then

Union:
$$A \cup B = \{1, 2, 3, 4, 5, 7\}$$

Intersection:
$$A \cap B = \{5\}$$

Complement:
$$B^c = \{1, 3, 6, 7, 8\}$$

Disjoint if
$$A \cap D = \{\varnothing\}$$

Exhaustive if $A \cup B \cup D = \mathcal{S}$

D.2 Distributive law of unions and intersections

Unions and intersections can be distributed:

$$A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$$

$$A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$$

D.3 DeMorgan's law

Distributive law of complement over union or intersection

$$(C_1 \cap C_2)^c = C_1^c \cup C_2^c$$

$$(C_1 \cup C_2)^c = C_1^c \cap C_2^c$$

$$(C_1 \cap C_2 \cap C_3)^c = C_1^c \cup C_2^c \cup C_3^c$$

$$(C_1 \cup C_2 \cup C_3)^c = C_1^c \cap C_2^c \cap C_2^c$$

$$\left(\bigcap_{k=1}^{\infty} C_k\right)^c = \bigcup_{k=1}^{\infty} C_k^c$$
$$\left(\bigcup_{k=1}^{\infty} C_k\right)^c = \bigcap_{k=1}^{\infty} C_k^c$$

D.4 Axioms of probability

P is a probability set function if

- 1. $P(A) \geq 0$, for all event $A \in \mathcal{B}$.
- 2. P(S) = 1.
- 3. If C_n is a sequence of disjoint events in \mathcal{B} , then

$$P\Big(\cup_{n=1}^{\infty} C_n\Big) = \sum_{n=1}^{\infty} P(C_n)$$

D.5 Probability Formulas

1.
$$P(A^c) = 1 - P(A)$$

2.
$$P(A) = P(A \cap B) + P(A \cap B^c)$$

3.
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4.
$$(A \cap B)^c = A^c \cup B^c$$

$$5. (A \cup B)^c = A^c \cap B^c$$

Inclusion-Exclusion (Can be extended)

DeMorgan's law

D.6 Ex: Project Funding

There are 3 projects that has applied for the grant. Let A_i represent an event that project i gets funded. There are 3 projects. Given

$$P(A_1) = .22, \quad P(A_2) = .25, \quad P(A_3) = .28$$

and

$$P(A_1 \cap A_2) = .11,$$
 $P(A_1 \cap A_3) = .05,$
 $P(A_2 \cap A_3) = .07,$ $P(A_1 \cap A_2 \cap A_3) = 0.01,$

Calculate the probability of :

- 1. P(At least one of project 1 and 2 get award)
- 2. P(Neither project 1 nor 2 get award)
- 3. P(Only project 3 is awarded)

D.7 Ex: Lab (work vs Referral)

D.8 Inc-Exc Extended