3B Expectation

Contents

3B The Expected Value (Theoretical Mean)

[ToC]

B.1 Intro

- Expected Value of a random variable X, whose range is $x_1, x_2, x_3, \ldots x_n$ is defined as

$$E(X) = \mu = \sum_{i=1}^{n} x_i \cdot p(x_i)$$

B.2 Ex: Throw a Die Once

pmf is:

x	1	2	3	4	5	6
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Then the expectation is

$$E(X) = 1 \cdot \left(\frac{1}{6}\right) + 2 \cdot \left(\frac{1}{6}\right) + 3 \cdot \left(\frac{1}{6}\right) + 4 \cdot \left(\frac{1}{6}\right) + 5 \cdot \left(\frac{1}{6}\right) + 6 \cdot \left(\frac{1}{6}\right) = 3.5$$

So what does this number mean?

B.3 Expectation is a long-run average

Suppose you rolled a die 20 times.

```
{ 4 3 1 4 1 3 5 3 2 6 4 1 5 1 6 6 3 4 4 6 }
```

Calculate the sample mean:

```
(4+3+1+4+1+3+5+3+2+6+4+1+5+1+6+6+3+4+4+6) / 20
```

That's same as...

The sample mean:

```
(1+1+1+1+2+3+3+3+3+4+4+4+4+4+5+5+6+6+6+6) / 20
```

That's same as...

$$\bar{X} = 1 \cdot \left(\frac{4}{20}\right) + 2 \cdot \left(\frac{1}{20}\right) + 3 \cdot \left(\frac{4}{20}\right) + 4 \cdot \left(\frac{5}{20}\right) + 5 \cdot \left(\frac{2}{20}\right) + 6 \cdot \left(\frac{4}{20}\right)$$

Sample Mean

$$\begin{aligned}\bar{X} &= 1 \cdot \left(\text{Rel. Freq. of \#1}\right) + 2 \cdot \left(\text{Rel. Freq. of \#2}\right) + 3 \cdot \left(\text{Rel. Freq. of \#3}\right) \\ &+ \quad 4 \cdot \left(\text{Rel. Freq. of \#4}\right) + 5 \cdot \left(\text{Rel. Freq. of \#5}\right) + 6 \cdot \left(\text{Rel. Freq. of \#6}\right)\end{aligned}$$

B.4 Theoretical Mean (Expectation)

Theoretical Mean (Expectation)

$$E(X) = 1 \cdot \left(\frac{1}{6}\right) + 2 \cdot \left(\frac{1}{6}\right) + 3 \cdot \left(\frac{1}{6}\right) + 4 \cdot \left(\frac{1}{6}\right) + 5 \cdot \left(\frac{1}{6}\right) + 6 \cdot \left(\frac{1}{6}\right) = 3.5$$

- Sample Mean converges to The Expectation as $n \to \infty$.

B.5 Ex: Pooled Blood Testing

- Each blood sample has .1 change of testing positive.
- New procedure called ”pooled testing” combines 10 blood sample before testing.
- If comes back negative, no further test is done. If comes back positive, then 10 more test must be done using individual samples.
- What is the long-run average of the test number in the new scheme?

B.6 Ex: Casino Simplified

- You charge each person \$1 to play this game.
- Each player has 1% chance of winning \$100.
- Does this make sence as business?
- What if you change 1% chance to .5% chance?

B.7 How to calculate $E(g(X))$

- Expected Value of a function of random variable X, say $g(X)$ is defined as

$$E\big(g(X)\big) = \sum_{i=1}^{n} g(x_i) \cdot p(x_i)$$

- If a and b are constants, then

$$E(aX + b) = aE(X) + b$$

B.8 Ex: Expectation of $g(X)$

x	1	2	3
$P(X = x)$	.5	.1	.4

- $E(X)$ is 1.9.
- Calculate $E(X^2)$.
- Calculate $E(3X + 5)$