4A ContRV

Contents

4A Preliminaries

[ToC]

A.1 Continuous Random Variable

- r.v. whose range is a interval on a real line or a disjoint union of such intervals.
- Suppose X is a r.v. which takes any value within the interval $[0, 1]$ with equal probability.

  What value can we assign to $P(X = .5)$?

A.2 For any continuous RV $P(X = c)$ is zero

- For any continuous R.V. X,

$$P(X = c) = 0 \qquad \text{for any constant } c.$$

- This means that we now have pdf instead of pmf.

Discrete RV						
X	1	2	3	4	5	6
pmf	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A.3 Probability density function (pdf) instead of pmf

pdf of continuous r.v. X is a functon $f(x)$ such that for any two numbers a and b with $a \leq b$,

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

Pdf must satisfy:

1. $f(x) \geq 0$ for all x.
2. $\int_{-\infty}^{\infty} f(x)dx = 1$.

Now the AREA UNDER PDF is the probability.

A.4 Cumulative Distribution Function (CDF) is still the same

of r.v. X is a functon $F(x)$ defined for every number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx$$

If X is a continuous r.v. with pdf $f(x)$ and cdf $F(x)$ then at every x at which the derivative $F'(x)$ exists,

$$F'(x) = f(x).$$

Cdf must satisfy:

1. $F(-\infty) = 0$ and $F(\infty) = 1$.
2. non-decreasing.
3. right continuous.

For any number a and b with $a < b$,

$$P(X > a) = 1 - F(a)$$

$$P(a \leq X \leq b) = F(b) - F(a)$$

A.5 Percentiles

Let p a number between 0 and 1. The $(100 \times p)$th percentile of the distribution of a continuous r.v. X, denoted η_p, is a number such that

$$F(\eta_p) = p$$

A.6 Example:

Let rv X have pdf

$$f(x) = \begin{cases} Kx^2 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

1. What is K?
2. What is $P(1.5 \leq X \leq 2)$?
3. What is $F(x)$?
4. What is 70th percentile of X?

What is $P(1.5 \leq X \leq 2)$?

What is $F(x)$?

What is 70th percentile of X?

A.7 Expected Values

Expected or mean value of a continuous r.v. X with pdf f(x) is

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx.$$

If $h(\cdot)$ is any function, then

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x) dx.$$

Therefore,

$$E(h(X)) = h\Big(E(x)\Big)$$

if $h(\cdot)$ is a linear function. In other words, $E(aX + b) = aE(X) + b$. $\square$

A.8 Variance

Variance of a continuous r.v. X with pdf f(x) is

$$V(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = E\Big[(x-\mu)^2\Big]$$

and standard deviation (SD) of X is

$$\sigma = \sqrt{\sigma^2}.$$

A.9 Example:

Let rv X have pdf

$$f(x) = \begin{cases} Kx^2 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

1. What is $E(X)$?
2. What is $V(X)$?

What is $V(X)$?

A.10 Uniform Distribution

- pdf

$$f(x) = \frac{1}{B-A} \qquad \text{for } A \leq x \leq B$$

  and 0 otherwise.

- CDF

$$F(x) = P(X \leq x) = \frac{x-A}{B-A}.$$

- Expectation and Variance:

$$E(X) = \frac{B+A}{2} \qquad V(X) = \frac{(B-A)^2}{12}$$