4A ContRV

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4A Preliminaries

[ToC]

A.1 Continuous Random Variable

- r.v. whose range is a interval on a real line or a disjoint union of such intervals.
- \bullet Suppose X is a r.v. which takes any value within the interval [0,1] with equal probability.

What value can we assign to P(X = .5)?

A.2 For any continuous RV P(X = c) is zero

• For any continuous R.V. X,

$$P(X = c) = 0$$
 for any constant c .

• This means that we now have pdf instead of pmf.

Discrete RV						
\overline{X}	1	2	3	4	5	6
pmf	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A.3 Probability density function (pdf) instead of pmf

pdf of continuous r.v. X is a function f(x) such that for any two numbers a and b with $a \leq b$,

$$P(a \le X \le b) = \int_a^b f(x)dx$$

Pdf must satisfy:

- 1. $f(x) \ge 0$ for all x.
- $2. \int_{-\infty}^{\infty} f(x) dx = 1.$

Now the AREA UNDER PDF is the probability.

A.4 Cumulative Distribution Function (CDF) is still the same

of r.v. X is a function F(x) defined for every number x by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$$

If X is a continuous r.v. with pdf f(x) and cdf F(x) then at every x at which the derivative F'(x) exists,

$$F'(x) = f(x).$$

Cdf must satisfy:

- 1. $F(-\infty) = 0$ and $F(\infty) = 1$.
- 2. non-decreasing.
- 3. right continuous.

For any number a and b with a < b,

$$P(X > a) = 1 - F(a)$$

$$P(a \le X \le b) = F(b) - F(a)$$

A.5 Percentiles

Let p a number between 0 and 1. The $(100 \times p)$ th percentile of the distribution of a continuous r.v. X, denoted η_p , is a number such that

$$F(\eta_p) = p$$

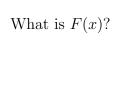
A.6 Example:

Let rv X have pdf

$$f(x) = \begin{cases} Kx^2 & \text{if } 1 \le x \le 3\\ 0 & \text{otherwise} \end{cases}$$

- 1. What is K?
- 2. What is $P(1.5 \le X \le 2)$?
- 3. What is F(x)?
- 4. What is 70th percentile of X?

What is $P(1.5 \le X \le 2)$?



What is 70th percentile of X?

A.7 Expected Values

Expected or mean value of a continuous r.v. X with pdf f(x) is

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx.$$

If $h(\cdot)$ is any function, then

$$E(h(X)) = \int_{-\infty}^{\infty} h(x)f(x)dx.$$

Therefore,

$$E(h(X)) = h\Big(E(x)\Big)$$

if $h(\cdot)$ is a linear function. In other words, E(aX + b) = aE(X) + b.

A.8 Variance

Variance of a continuous r.v. X with pdf f(x) is

$$V(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = E[(x - \mu)^2]$$

and standard deviation (SD) of X is

$$\sigma = \sqrt{\sigma^2}$$
.

A.9 Example:

Let rv X have pdf

$$f(x) = \begin{cases} Kx^2 & \text{if } 1 \le x \le 3\\ 0 & \text{otherwise} \end{cases}$$

- 1. What is E(X)?
- 2. What is V(X)?

What is V(X)?

A.10 Uniform Distribution

 \bullet pdf

$$f(x) = \frac{1}{B - A}$$
 for $A \le x \le B$

and 0 otherwise.

• CDF

$$F(x) = P(X \le x) = \frac{x - A}{B - A}.$$

• Expectation and Variance:

$$E(X) = \frac{B+A}{2}$$
 $V(X) = \frac{(B-A)^2}{12}$