4B Normal

Contents

3B Normal Distribution

[ToC]

B.1 Normal Distribution

- pdf for $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

- CDF

$$F(X) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$$

but this is analytically non-tractable, and must be evaluated numerically. We have a table for the case $(\mu, \sigma^2) = (0, 1)$.

- Mean and Variance

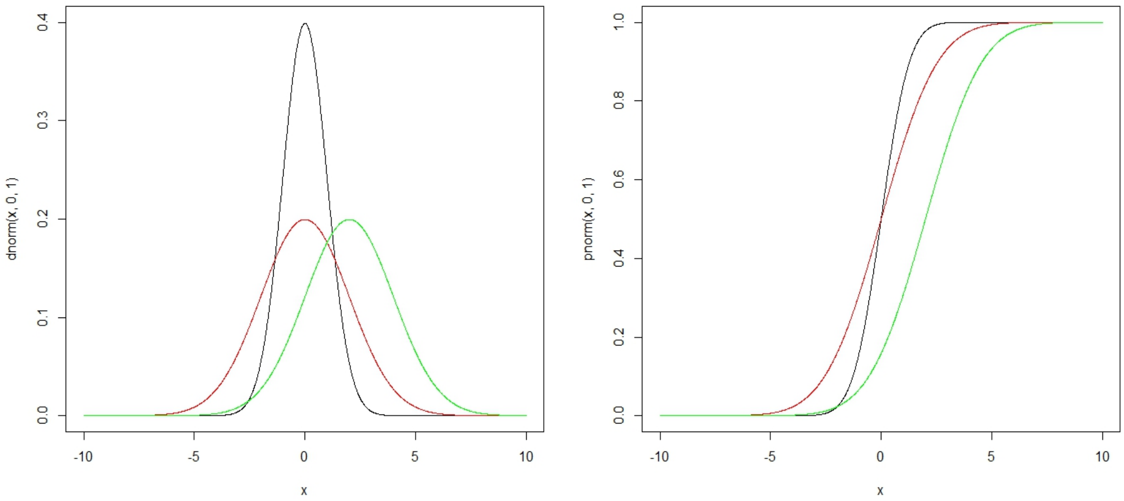
$$E(X) = \mu \qquad V(X) = \sigma^2$$

B.2 TI-84 for Normal(μ, σ)

```
2nd -> VARS (Same as DISR)

normalpdf(x, mu, sigma)           # f(x)
normalcdf(a, b, mu, sigma)        # F(b) - F(a)
invNormal(p, mu, sigma)           # F(x) = p
```

B.3 $\mathbf{N}(\mu = 0, \sigma = 1)$, $\mathbf{N}(\mu = 0, \sigma = 2)$ and $\mathbf{N}(\mu = 2, \sigma = 2)$



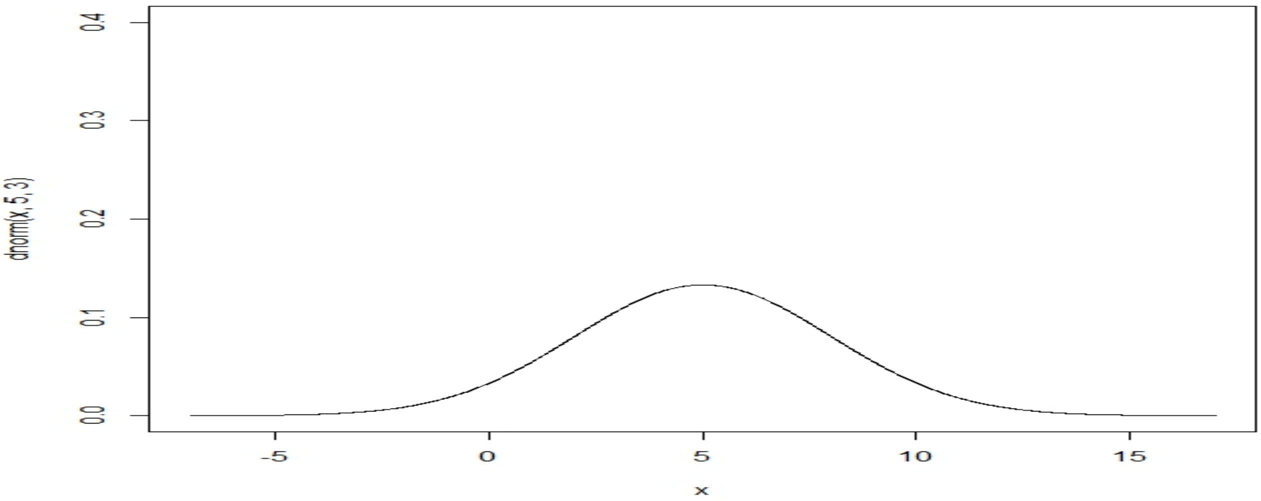
B.4 Empirical Rule

In $X \sim \mathrm{N}(\mu, \sigma^2)$, then

1. with probability .68, X is within 1 SD away from μ.
2. with probability .95, X is within 2 SD away from μ.
3. with probability .99.7, X is within 3 SD away from μ.

```
x <- seq(-7,17,.01)
plot(x, dnorm(x, 5, 3), type="l", ylim=c(0,.4))
```

B.5 $\mathbf{N}(\mu = 5, \sigma^2 = 3^2)$



B.6 Standard Normal Distribution

- N(0,1) is called Standard Normal Distribution.
- Pdf of standard normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

- CDF

$$F(t) = P(Z \leq t) = \Phi(t).$$

- Table A.3 in the textbook lists values of $\Phi(t)$.

B.7 z_α Notation

- Z is used to denote Standard Normal random variable.

- z_α denotes $(1-\alpha)100$ th percentile of Z.

- i.e. $z_{.05}$ = [95th percentile of Z]

B.8 Using Normal Table

- Find $P(Z \leq 1.4)$

- Find $P(Z > .53)$

- Find 90th percentile of Z

- Find $Z_{.05}$

B.9 Standardization of Normal:

$X \sim \mathrm{N}(\mu, \sigma^2)$ $Z \sim \mathrm{N}(0,\ 1)$

$$Z = \frac{X - \mu}{\sigma} \implies$$

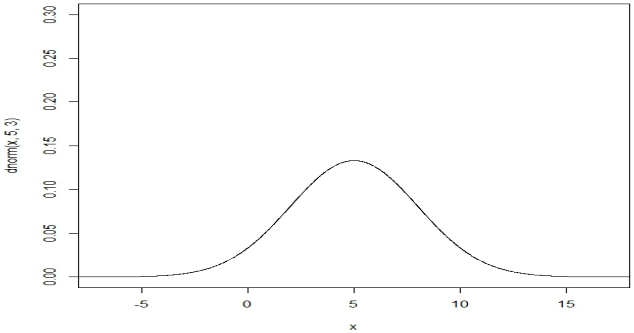
$$\impliedby \quad X = \mu + Z\sigma$$

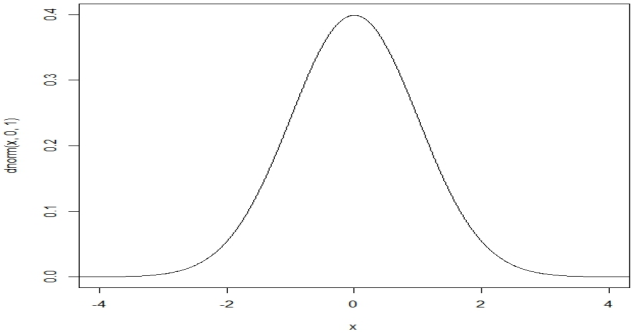
B.10 Use Standardization to find $F(x)$

Using standardization, you can use $\Phi(\cdot)$ to figure out the cdf of X.

$$P(X \le a) = P\Big(\frac{X-\mu}{\sigma} \le \frac{a-\mu}{\sigma}\Big) \quad = \quad \Phi\Big(\frac{a-\mu}{\sigma}\Big)$$

Find $P(X \le 8)$ in $\mathrm{N}(5, 3^2)$.





- $$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a) \quad = \quad \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

- $$P(X > a) = 1 - P(X \leq a) = 1 - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

- $P(X \geq a)$

- $P(X = a)$

B.11 Example: Tree Height

Diameter at breast height (in.) of trees of certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$.

1. What is probability that randomly chosen tree has diameter less than 10in?
2. What is probability that randomly chosen tree has diameter greater than 20in?
3. What is probability that randomly chosen tree has diameter between 5 and 15?
4. What is range of diameter represents the middle 68% of the trees?

$X \sim N(8.8, 2.8^2)$

What is probability that randomly chosen tree has diameter greater than 20in?

$$X \sim N(8.8, 2.8^2)$$

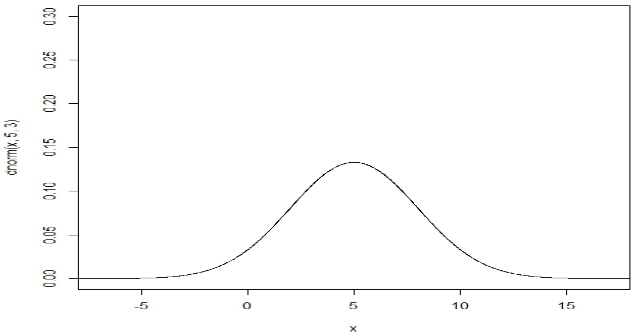
What is probability that randomly chosen tree has diameter between 5 and 15?

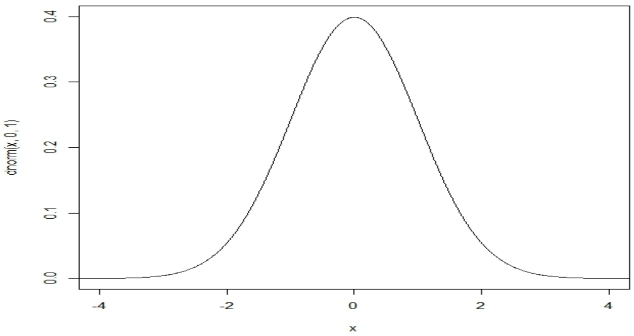
$$X \sim N(8.8, 2.8^2)$$

What is range of diameter represents the middle 68% of the trees?

B.12 Finding percentile of $N(\mu, \sigma^2)$

Find 90th percentile of $N(5, 3^2)$.





B.13 Ex: Find Percentile

Suppose X is Normal random variable with $\mu = 5$ and $\sigma = 2$. What is the 70th percentile of X?

B.14 Ex: Find Percentile 2

Suppose X is a Normal random variable with μ and $\sigma = 2$. For what value of μ, the 70th percentile of X equal to 3.5?

B.15 Ex: Tree Height 2

Diameter at breast height (in.) of trees of certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$.

1. To protect younger tree from being cut, we want to ban cutting of smallest 70% of the trees. For what diameters should we ban the cutting?

2. For what value of c does interval $(8.8 \pm c)$ contain 95% of diameters?

B.16 Ex: Cereal Box

Cereal box is being filled at a factory. Box says it contains 32oz. Let the machine to have $\sigma^2 = 2$ and define [underfilled] as Box< 30, [overfilled] as Box> 33.

1. Determine μ if we want $P(\text{underfilled}) = .03$?
2. For that μ, what is $P(\text{overfilled})$?
3. For the same μ, what σ is needed so that $P(\text{overfilled}) = .05$?

2 For that μ, what is $P(\text{overfilled})$?

3 For the same μ, what σ is needed so that $P(\text{overfilled}) = .05$?

B.17 Normal Approximation of Binomial

- If n is sufficiently large ($np \geq 10$ and $n(1-p) \geq 10$),

$$\text{Binomal}(n, p) \quad \approx \quad \text{Normal}(np, np(1-p))$$

- Continuity correction of binomial approximation is done by the formula

$$P(X \leq x) = \Phi\left(\frac{x + .5 - np}{\sqrt{np(1-p)}}\right).$$