4C Exp

Contents

4C Exponential Distribution

[ToC]

C.1 Exponential Distribution

- pdf of Exp(λ)

$$f(x) = \lambda e^{-\lambda x} \qquad \text{for } x \geq 0$$

- CDF

$$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$$

- Mean and Variance

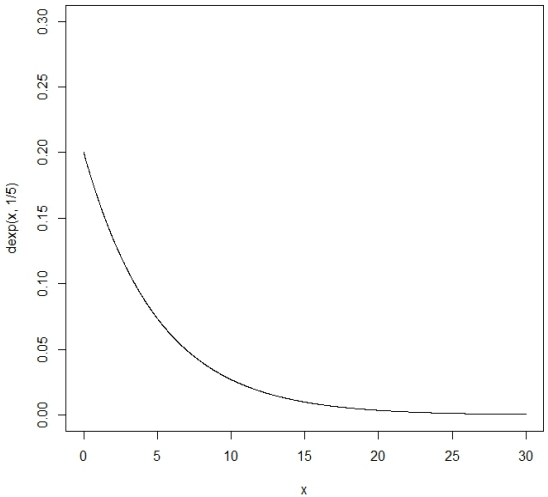
$$E(X) = 1/\lambda \qquad V(X) = 1/\lambda^2$$

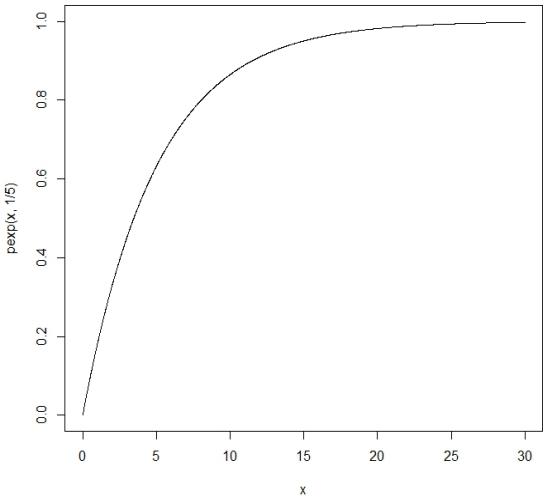
C.2 R code for Exponential(λ)

```
dexp(.5, 1/5)          #- F(3):  CDF
pexp(.5, 1/5)          #- p(3):  pmf


layout( matrix(1:2, 1, 2) )

x <- seq(0,30,.01)
plot(x, dexp(x, 1/5), type="l", ylim=c(0,.3))   #- PMF plot -
plot(x, pexp(x, 1/5), type="l", ylim=c(0,1))   #- CDF plot -
```


dexp(x, 1/5)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
5
10
15
20
25
30
x


pexp(x, 1/5)
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
x

C.3 CDF of exponential

$$
\begin{aligned}
F(x;\lambda) &= P(X \leq x) \\
&= \int_0^x \lambda e^{-\lambda y} dy \\
&= -\tfrac{\lambda}{\lambda} e^{-\lambda y} \Big|_0^x \\
&= 1 - e^{-\lambda x}.
\end{aligned}
$$

C.4 Mean

If $X \sim \text{Exp}(\lambda)$, then

$$\begin{aligned} E(X) &= \int_0^\infty x \cdot \lambda e^{-\lambda x} dx \\ &\qquad\qquad \text{Integraing by parts,} \\ &= -x\lambda e^{-\lambda x}\Big|_0^\infty + \int_0^\infty e^{-\lambda x} dx \\ &= 0 - \tfrac{1}{\lambda} e^{-\lambda x}\Big|_0^\infty \\ &= \tfrac{1}{\lambda} \end{aligned}$$

C.5 Variance

If $X \sim \text{Exp}(\lambda)$, then

$$V(X) = \frac{1}{\lambda^2}.$$

$$\begin{aligned} E(X^2) &= \int_0^\infty x^2 \cdot \lambda e^{-\lambda x} dx \\ &\qquad\qquad \text{Integraing by parts,} \\ &= -x^2 \lambda e^{-\lambda x}\Big|_0^\infty + \int_0^\infty 2x e^{-\lambda x} dx \\ &= 0 + \tfrac{2}{\lambda} \int_0^\infty x \lambda e^{-\lambda x} dx \\ &= 0 - \tfrac{2}{\lambda} E(X) \quad = \quad \tfrac{2}{\lambda^2} \end{aligned}$$

Therefore,

$$V(X) \;= E(X^2) - \Big(E(X)\Big)^2 \quad = \quad \tfrac{1}{\lambda^2} - \Big(\tfrac{1}{\lambda}\Big)^2 \quad = \quad \tfrac{1}{\lambda^2}.$$

C.6 Ex: Two Servers

- Suppose there are two servers, A and B.
- Their service waiting time is exponentially distributed with mean of 3 sec. Assume independence between the waiting time of two servers.
- Your task was sent to Server A. What is the probability that your task has to wait more than 7 sec?
- Your task was sent to both Server A and B, so that it will be served by whichever the server that opens up first. What is the probability that your task has to wait more than 7 sec?

C.7 Ex: Lifetime

- Suppose each appliance's lifetime can be modeled by exponential distributed with mean of 2 years.
- What is the probability that
- Suppose you sold 10 appliance in the same day. What is the probability that more than 4 of them will break within 3 years?

C.8 Ex: Half Life of C14

About 5700 years.

Which means lambda is about .0001216