6A CI for Mean

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6A Sample Mean and Confidence Interval

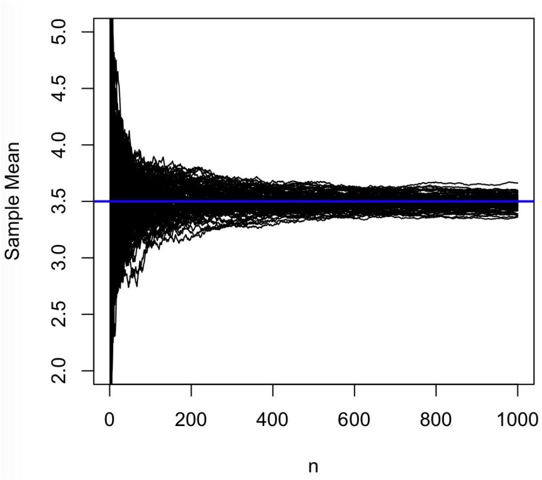
[ToC]

A.1 Central Limit Theorem

- When $X_1, \ldots, X_n$ are random sample from an experiment with mean μ and SD σ,

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

1. When $X \sim N(\mu, \sigma)$, then above result is exact, for any n.
2. When $n > 40$, X can have any distribution, and above result holds approximately.



X

(each experiment)

$\overline{X}$

$\{X_1, X_2, \ldots, X_n\}$

A.2 Three Scenarios

1. When $n > 40$. (Distribution of X doesn't matter. σ known or not doesn't matter)

2. When $n \leq 40$. (X has to be normally distributed)

   (a) σ is known.

   (b) σ is unknown and replaced with S.

A.3 Confidence Interval for μ

Suppose your data $X_1, \ldots, X_n$ are Random Sample from distributioin A with mean μ and variance σ^2. Now we know that $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$.

Then we know that

$$P(\bar{X} \text{ is within } \mu \pm z_{\alpha/2}\, \sigma/\sqrt{n}) = 1 - \alpha$$

Which is same thing as to say

$$P(\mu \text{ is within } \bar{X} \pm z_{\alpha/2}\, \sigma/\sqrt{n}) = 1 - \alpha.$$

Thus, our $100(1-\alpha)\%$ Confidence Interval for μ is

$$\overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

A.4 Ex: Hydro Turbines 1

- Old model of hydroelectric miniturbines averages 25.2 Kwatt output under the lab condition.
- Recently the model design was changed, and that supposed to improve the average output.
- Out of 50 units tested, sample mean 27.1, sigma is known to be 7.2.
- Is this result enough to claim the improvement on average?

A.5 One Sided CI

- $100(1-\alpha)\%$ two-sided Confidence Interval for μ is

$$\overline{X} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

- Take two-sided CI formula, change $\alpha/2$ to α.
- Pick one of the sign for upper-bound or lower-bound.

- $100(1-\alpha)\%$ One-sided upper-bound Confidence Interval for μ is

$$\left(-\infty, \quad \overline{X} + z_{\alpha}\frac{\sigma}{\sqrt{n}}\right)$$

- $100(1-\alpha)\%$ One-sided lower-bound Confidence Interval for μ is

$$\left(\overline{X} - z_{\alpha}\frac{\sigma}{\sqrt{n}}, \quad \infty\right)$$

A.6 Ex: Hydro Turbines 2

- Old model of hydroelectric miniturbines averages 25.2 Kwatt output under the lab condition.
- Recently the model design was changed, and that supposed to improve the average output.
- Out of 50 units tested, sample mean 27.1, sigma is known to be 7.2.
- Is this result enough to claim the improvement?

A.7 Use of CI

1. Want to show statistical evidence that $\mu = \mu_0$ is not plausible. If μ_o is not in CI, then reject.

2. Want to show statistical evidence that $\mu = \mu_0$ is plausible. Must have CI reasonablly small, then show that μ_0 is still in CI.

3. Want to show statistical evidence that $\mu > \mu_0$. Calculate lower-bound CI, and show that it's above μ_0.

4. Want to show statistical evidence that $\mu < \mu_0$. Calculate upper-bound CI, and show that it's above μ_0.

A.8 What Confidence means and doesn't mean

So what is the

$$P(\text{ true average output is between } (25.1\ 29.1))$$

A.9 When σ is unkown (n is still > 40)

Since we have $N > 40$ in this case, we just replace σ in the formula with S.

A.10 Ex: A/C unit lifetime 1

- Sample of 60 units have averaged in 3.72 years and standard deviation of 4.17.
- Calculate 95% two-sided lower-bound CI for true average lifetime.
- Calculate 95% one-sided lower-bound CI for true average lifetime.
- Is this an evidence that true average lifetime is more than 3 years?
- Is this an evidence that true average lifetime is more than 2.5 years?