6B Small N

Contents

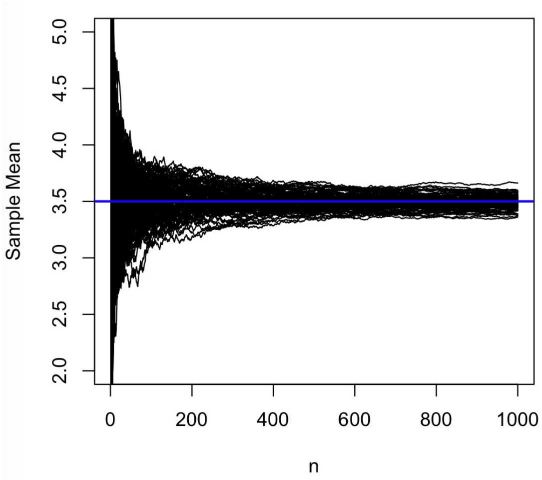
6B When we don't have a large sample size

[ToC]

- When n is not larger than 40, we must have $X \sim N(\mu, \sigma)$ to have the result

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

- σ may or may not be known.



X

(each experiment)

$\overline{X}$

$\{X_1, X_2, \ldots, X_n\}$

B.1 When σ is kown

- No change in formula.

B.2 Ex: Hydro Turbines 3

- Old model of hydroelectric miniturbines averages 25.2 Kwatt output under the lab condition.
- Recently the model design was changed, and that supposed to improve the average output.
- Out of 10 units tested, sample mean 27.1, sigma is known to be 7.2.
- Is this result enough to claim the improvement?

B.3 When σ is unknown

Suppose $X_1, \ldots, X_n$ be Random Sample from N(μ, σ) distribution, but now σ is unknown. Then we still have

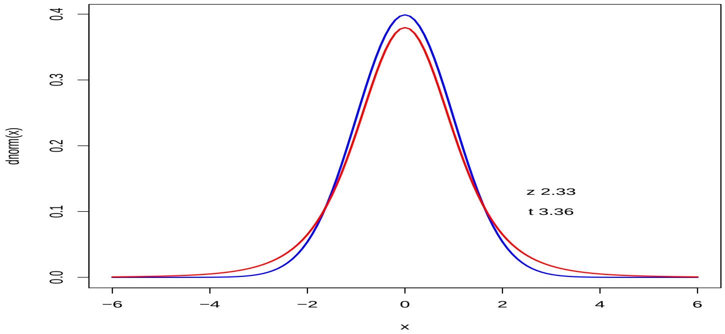
$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1).$$

But do not know σ. Then if we use S instead of σ, we have slightly different distribution,

$$\frac{\overline{X} - \mu}{S/\sqrt{n}} \sim t(n - 1).$$

Where $t(n - 1)$ is t-distribution with degrees of freedom $n - 1$.

B.4 Student's t-distribution



```
draw.t <- function(df=5, a=0){

  x <- seq(-6,6,.1)
  plot(x, dnorm(x), type='l', col="blue", lwd=2)
  lines(x, dt(x,df), col="red", lwd=2)

  if (a!=0) {
    c1<- qnorm(1-a);  c2 <- qt(1-a,df)
    text(3, .13, paste("z", round(c1,2)))
    text(3, .1, paste("t", round(c2,2)))
  }
}

draw.t(5)
draw.t(5, .05)
```

B.5 Confidence Interval with t-distribution

- $100(1-\alpha)\%$ (two-sided) Confidence Interval for μ is

$$\left(\overline{X} \pm t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}\right)$$

- For one-sided upper- or lower- CI, pick one of $+$ or $-$ sign above, and change $\frac{\alpha}{2}$ to α.

B.6 t-distribution on Calculators

```
#--- R
  dt(x, df)
  pt(x, df)
  qt(p, df)
```

```
#--- TI-84 (
  tpdf(x, df)
  tcdf(a, b, df)
  invT(p, df)
```

B.7 Ex: A/C unit lifetime 2

- Sample of 15 units have averaged in 3.72 years and sd of 4.17.
- Calculate 95% CI for true average of A/C units of same kind.

B.8 Ex: Tire Life

- The manufacturer of a new fiberglass tire claims that its average life will be at least 40,000 miles.
- To verify this claim a sample of 12 tires is tested, with their lifetimes (in 1,000s of miles) being as follows:
- Sample mean: 38.04. Sample SD: 2.49.

B.9 Ex: Heat Transfer

- An article in the Journal of Heat Transfer (Trans. ASME, Sec. C, 96. 1974. p. 59) described a new method of measuring the thermal conductivity of Armco iron.
- Using a temperature of 100F and a power input of 550 wtts, 10 measurements of thermal conductivity (in Btu/hr-ft-F) were obtained:
- $\bar{X} = 41.92$ $S = .284$