

# 7A CIforProp

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# A Sample Proportion and Confidence Interval

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## A.1 Central Limit Theorem

- When your random sample is binary (from a population with 0s and 1s), with  $p$ =overall proportion of 1s in the population. Then we let

$$X = [\text{number of 1s in the sample}], \quad \hat{p} = \frac{X}{n}$$

Then we have

$$X \sim \text{Bin}(n, p)$$

## A.2 CLT

1. If  $(np > 10)$  and  $(n(1 - p) > 10)$ , then Binomial can be approximated by Normal.
2.  $E(X) = np$  and  $V(X) = np(1 - p)$ .

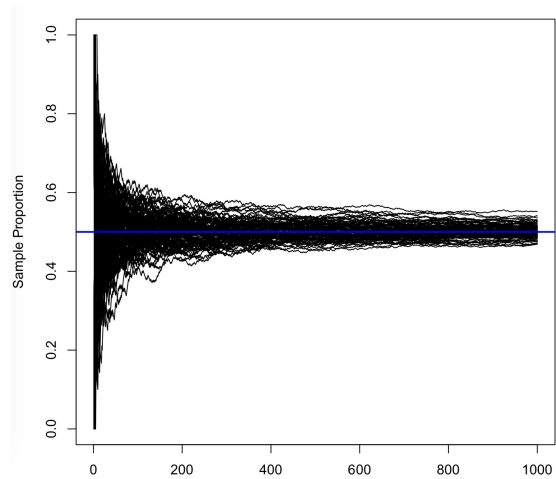
Then we have

$$X \sim \text{Bin}(n, p) \approx N\left(np, \sqrt{np(1 - p)}\right)$$

$$\hat{p} = \frac{X}{n} \approx N\left(p, \sqrt{\frac{p(1 - p)}{n}}\right)$$

## A.3 figure

0/1  
(each experiment)



$\hat{p}$

$\{0, 1, 0, 0, \dots, 1\}$

## A.4 Confidence Interval for $p$

Suppose your binary data are Random Sample from distribution with proportion  $p$ .

We know that  $\hat{p} \sim N\left(p, \sqrt{\frac{np(1-p)}{n}}\right)$ .

Then, our  $100(1 - \alpha)\%$  Confidence Interval for  $p$  is

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$

Similarly for one-sided CI.

## A.5 95% approx CI for $p$

Then, our  $100(1 - \alpha)\%$  Confidence Interval for  $p$  is

$$\hat{p} \pm 1.96 \sqrt{\frac{p(1-p)}{n}}$$

## A.6 Ex: Tire Share

Suppose that the Goodyear Tire company has historically held 42% of the market for automobile tires in US. Recent changes in company operation prompted the firm to test the validity of the assumption that it still controls 42% of the market. With  $n=100$ , sample showed 35/100 had Goodyear tires.

## A.7 Ex: Drought and Fertilizer Use

The percentage of farmers using fertilizers in an African country was known to be 35%. The drought and other events of the last few years are believed to have had a potential impact on the proportion of farmers using fertilizers. An international aid program wants to test if it changed. With  $n=550$ , sample proportion was  $= 242/550$ .

