7B TestOfHyp

Contents

A Test of Hypothesis

[ToC]

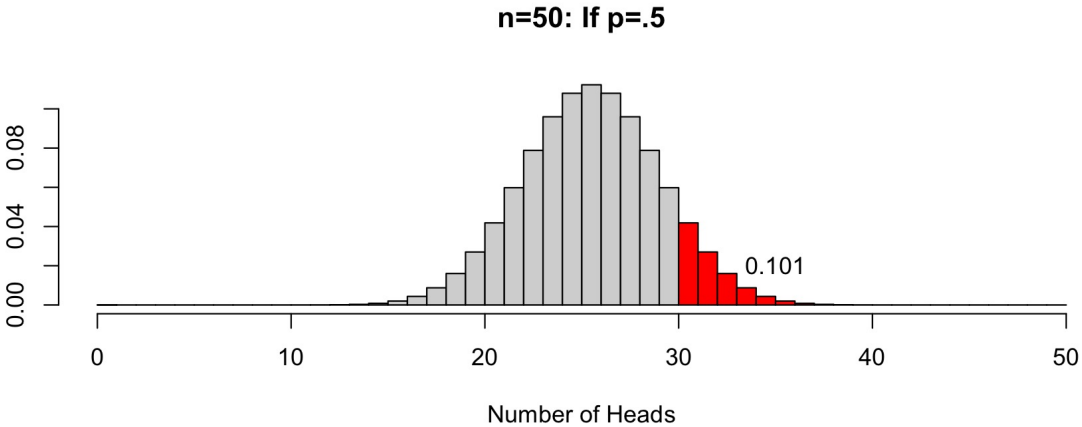
A.1 Example

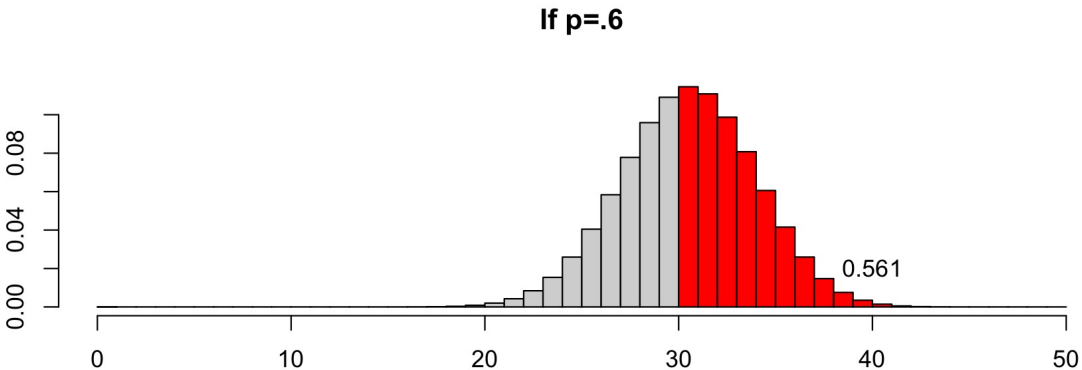
- Suppose we want to test two hypothesis

$$H_0 : \qquad p = .5$$
$$H_A : \qquad p = .6$$

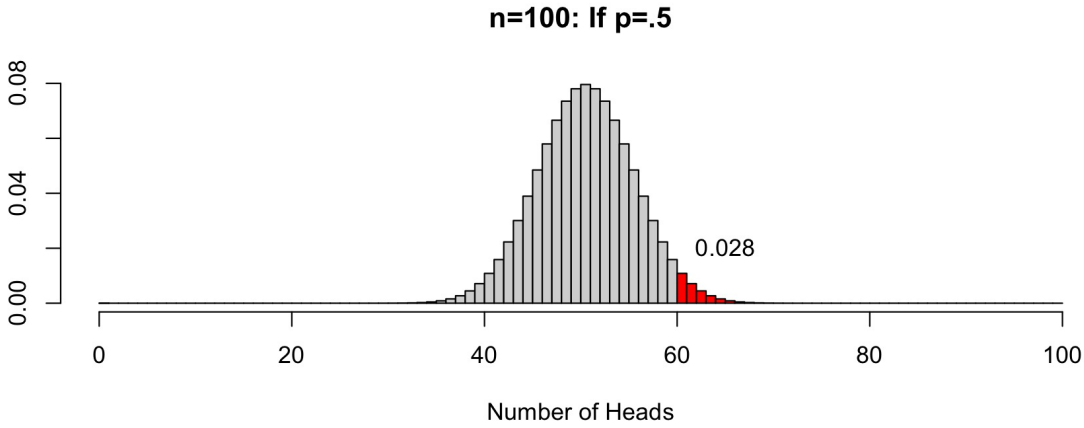
- How can we test these hypothesis?

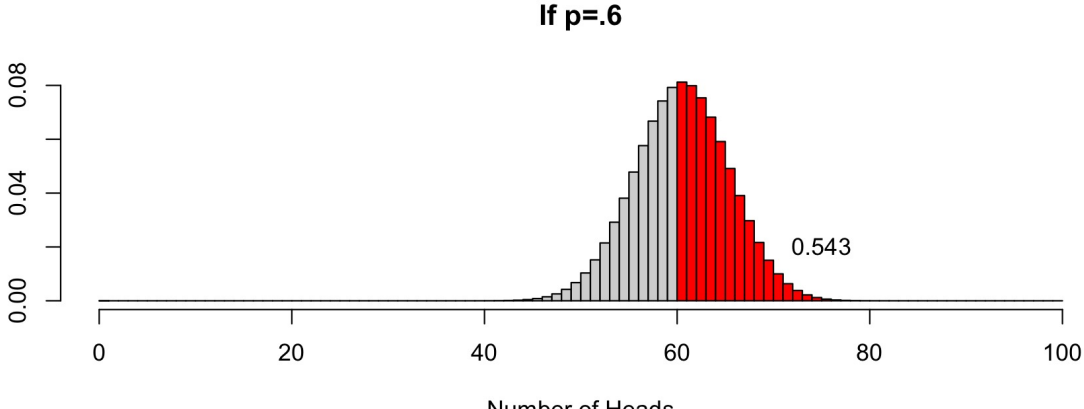
A.2 figure

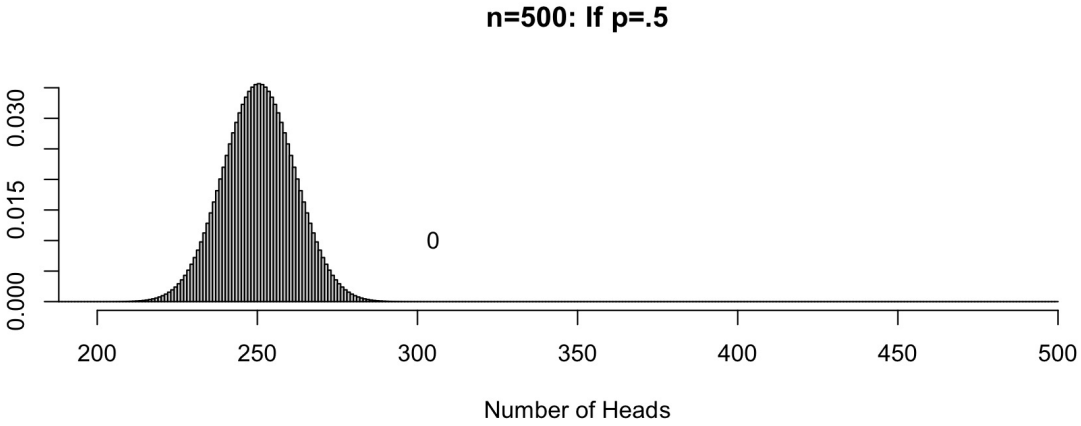

n=50: If p=.5
0.00
0.04
0.08
0.101
0
10
20
30
40
50
Number of Heads

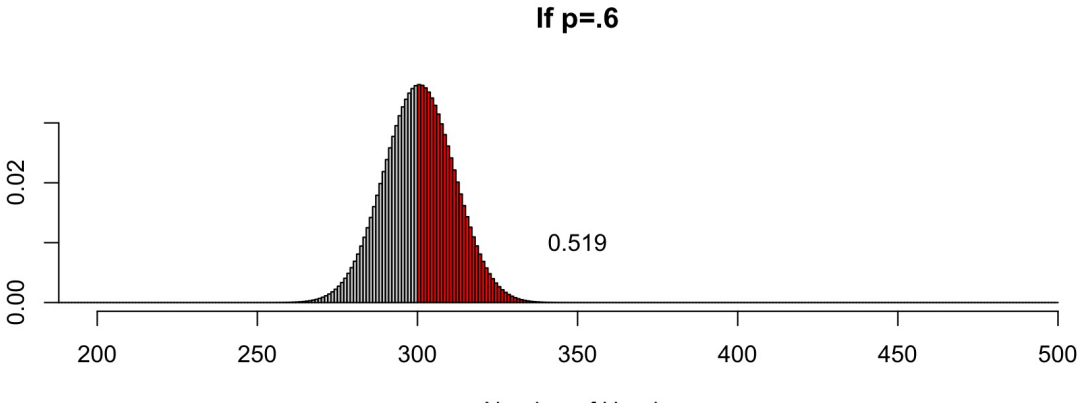

If p=.6
0.00
0.04
0.08
0.561
0
10
20
30
40
50

A.3 figure


n=100: If p=.5
0.08
0.04
0.00
0.028
0
20
40
60
80
100
Number of Heads


If p=.6
0.08
0.04
0.00
0.543
0
20
40
60
80
100


n=500: If p=.5
0.000
0.015
0.030
0
200
250
300
350
400
450
500
Number of Heads


If p=.6
0.00
0.02
0.519
200
250
300
350
400
450
500

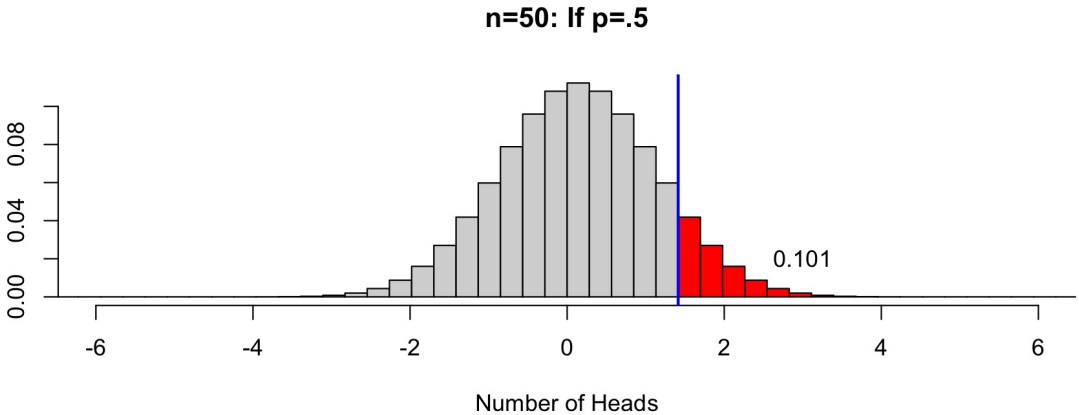
A.4 Standardize the statistic

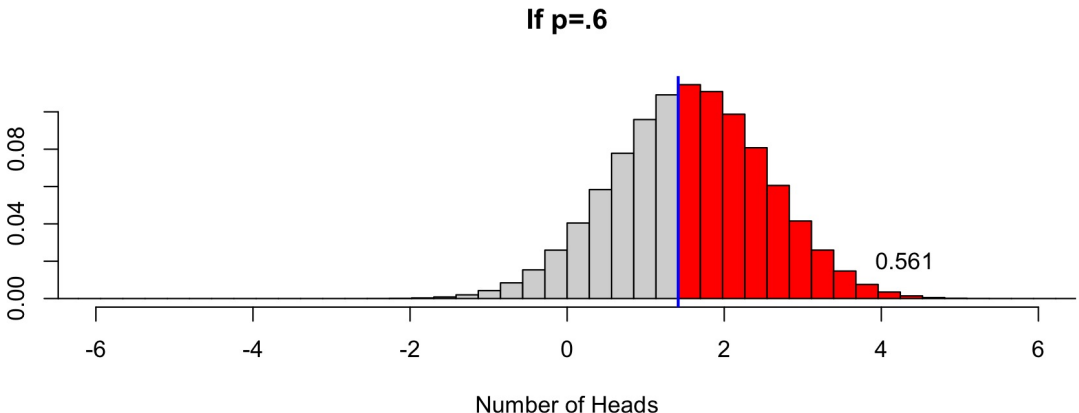
- Instead of looking at X, let's look at the z-score of $\hat{p} = X/n$.

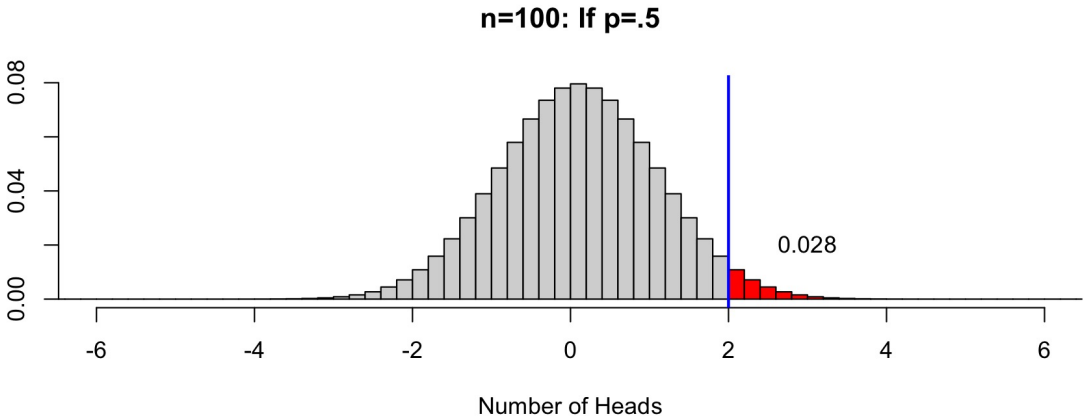
$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

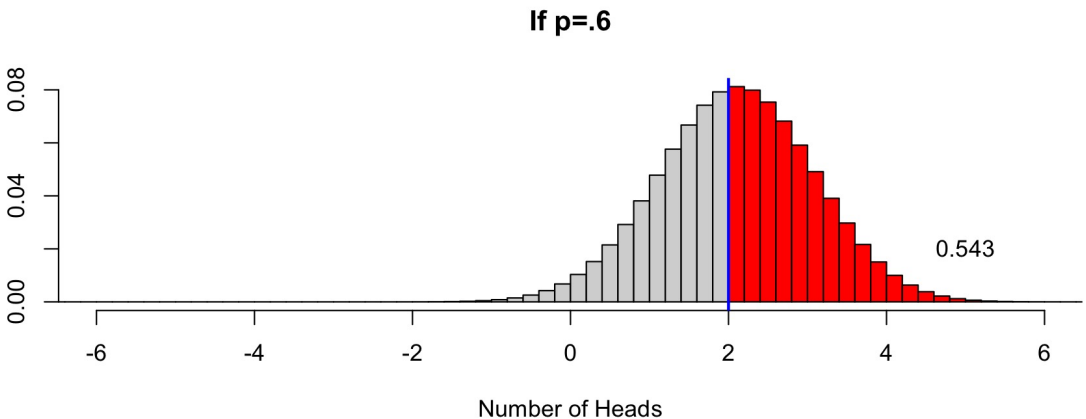
$$\frac{(.6 - .5)}{\sqrt{(.5)(.5)/50}} = 1.414 \qquad \frac{(.6 - .5)}{\sqrt{(.5)(.5)/100}} = 2 \qquad \frac{(.6 - .5)}{\sqrt{(.5)(.5)/500}} = 4.47$$

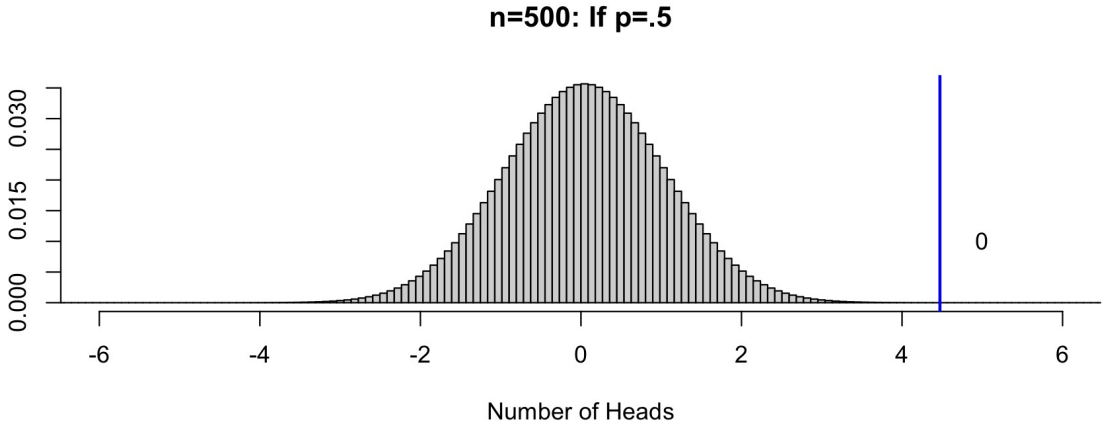
A.5 fig

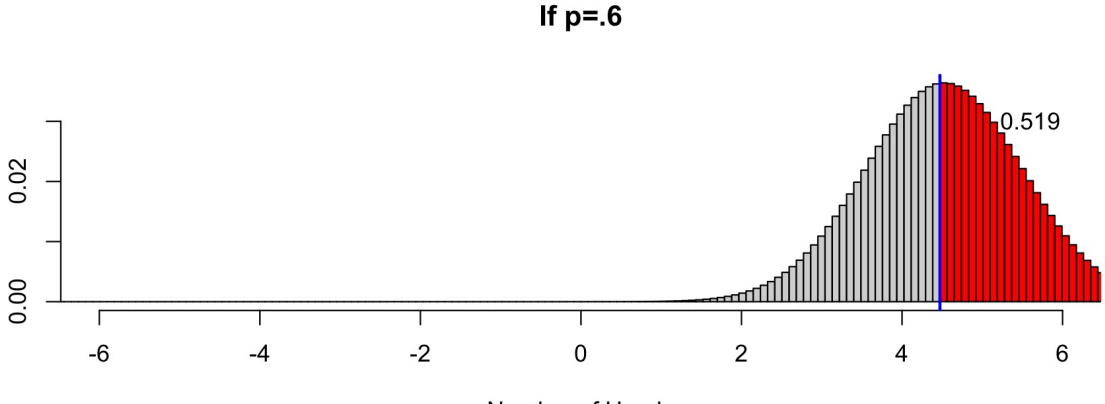

n=50: If p=.5
0.00
0.04
0.08
0.101
-6
-4
-2
0
2
4
6
Number of Heads


If p=.6
0.00
0.04
0.08
0.561
-6
-4
-2
0
2
4
6
Number of Heads


n=100: If p=.5
0.08
0.04
0.00
0.028
-6
-4
-2
0
2
4
6
Number of Heads


If p=.6
0.08
0.04
0.00
0.543
-6
-4
-2
0
2
4
6
Number of Heads

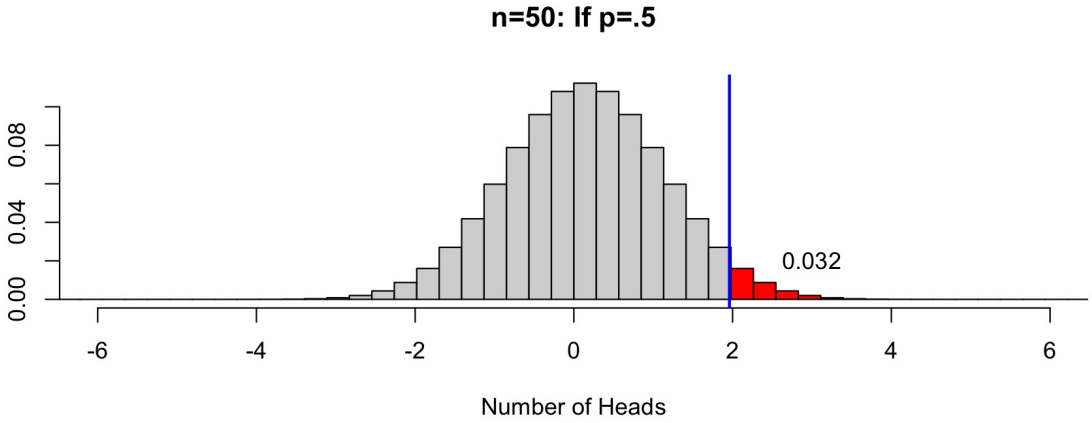

n=500: If p=.5
0.000
0.015
0.030
0
-6
-4
-2
0
2
4
6
Number of Heads

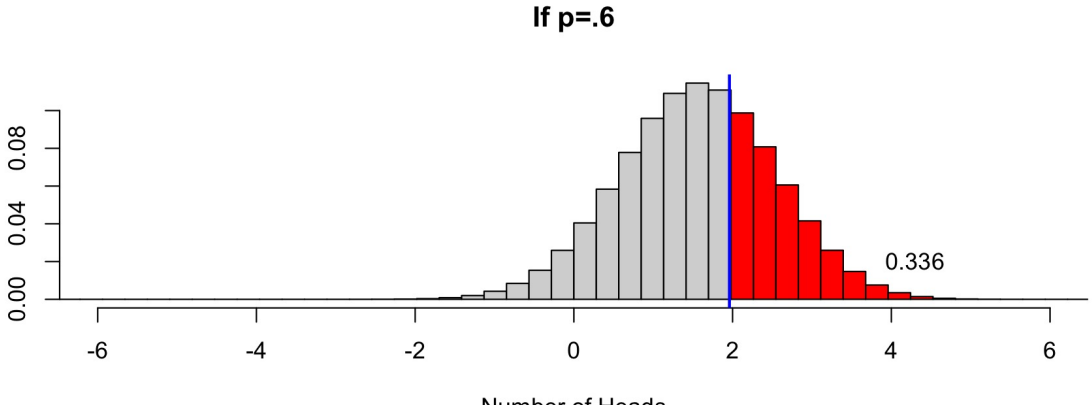

If p=.6
0.00
0.02
0.519
-6
-4
-2
0
2
4
6

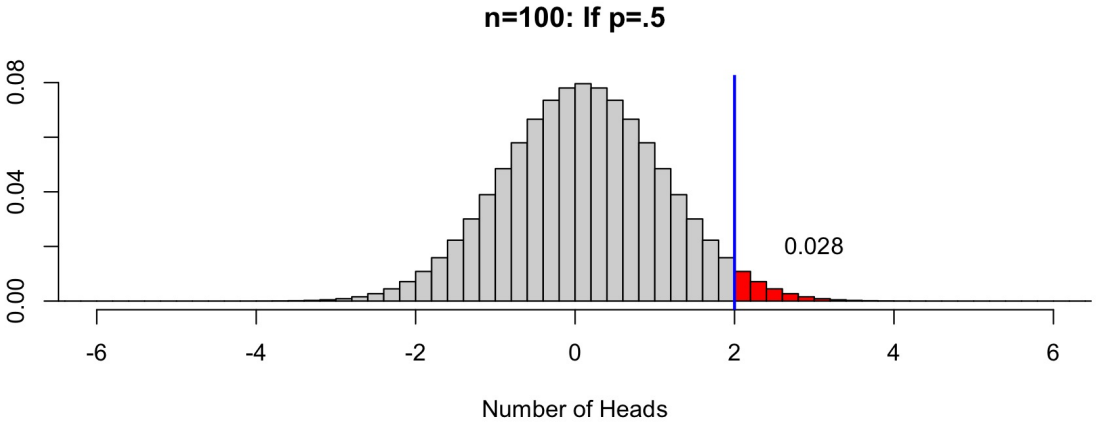
A.6 Critical Value

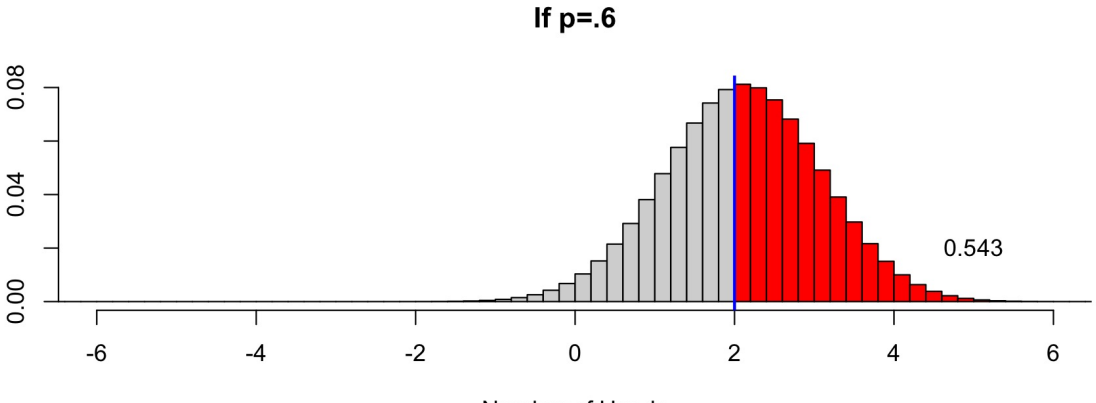
- Now that the distribution is fixed under H_0, we can fix the critical value (blue line) as well:

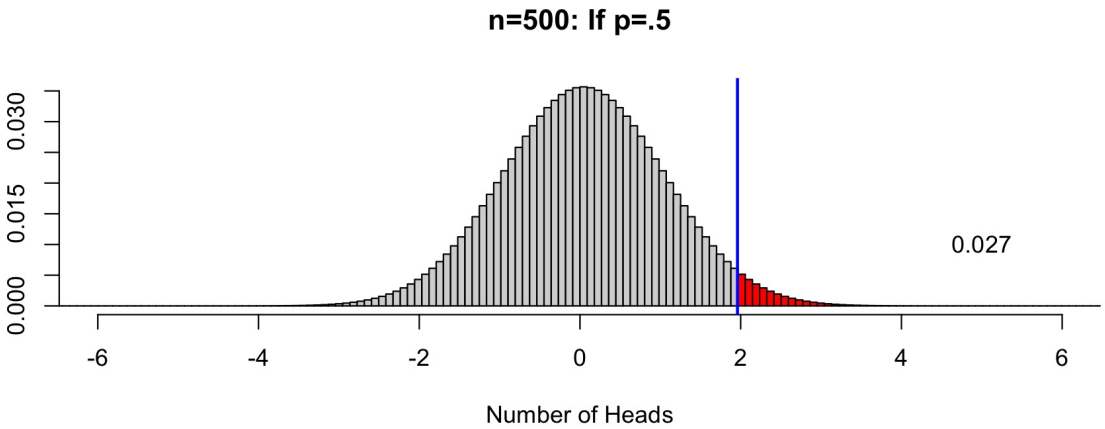
A.7 fig

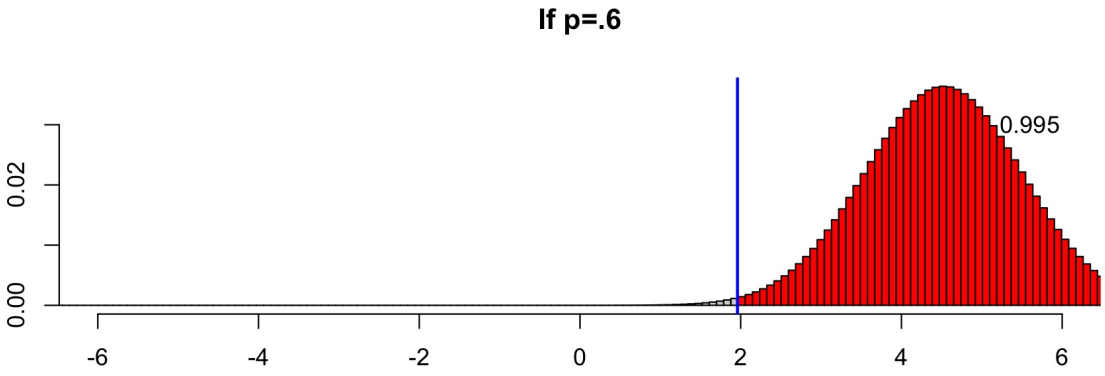

n=50: If p=.5
0.00
0.04
0.08
0.032
-6
-4
-2
0
2
4
6
Number of Heads


If p=.6
0.00
0.04
0.08
0.336
-6
-4
-2
0
2
4
6


n=100: If p=.5
0.08
0.04
0.00
0.028
-6
-4
-2
0
2
4
6
Number of Heads

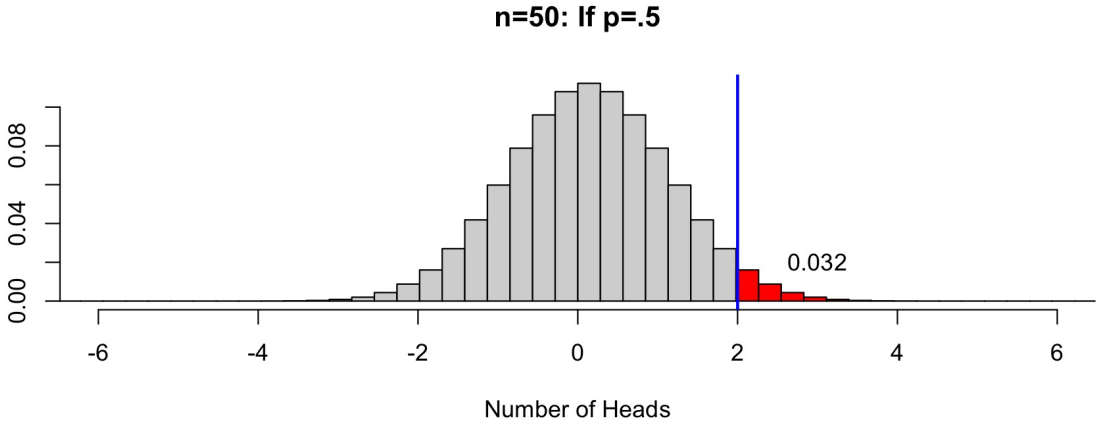

If p=.6
0.08
0.04
0.00
0.543
-6
-4
-2
0
2
4
6

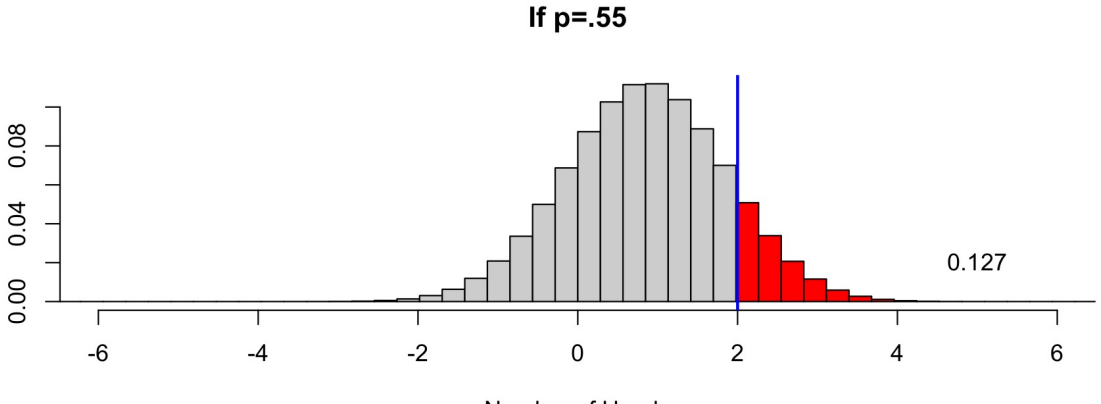

n=500: If p=.5
0.000
0.015
0.030
0.027
-6
-4
-2
0
2
4
6
Number of Heads

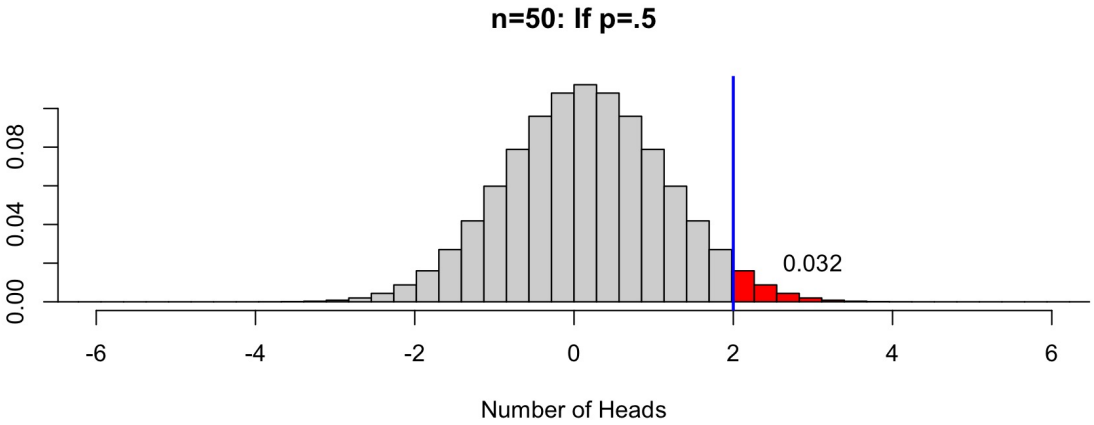

If p=.6
0.00
0.02
0.995
-6
-4
-2
0
2
4
6

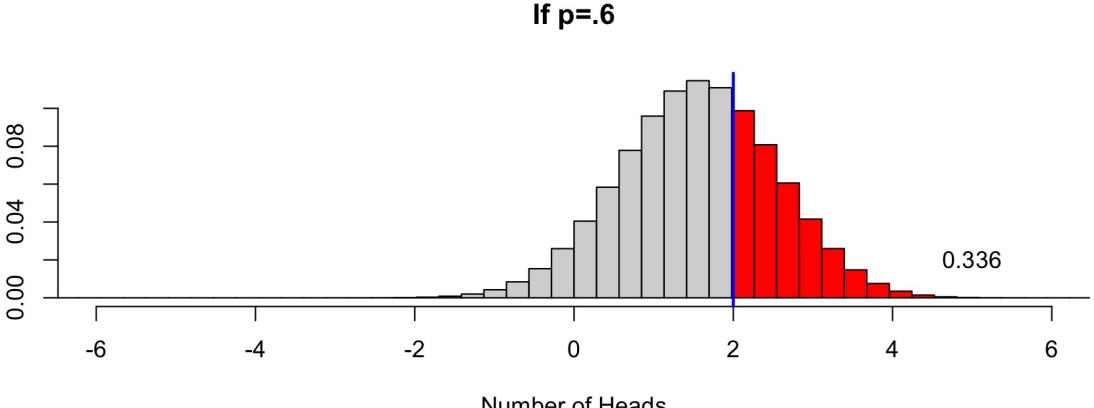
A.8 Under Alternative

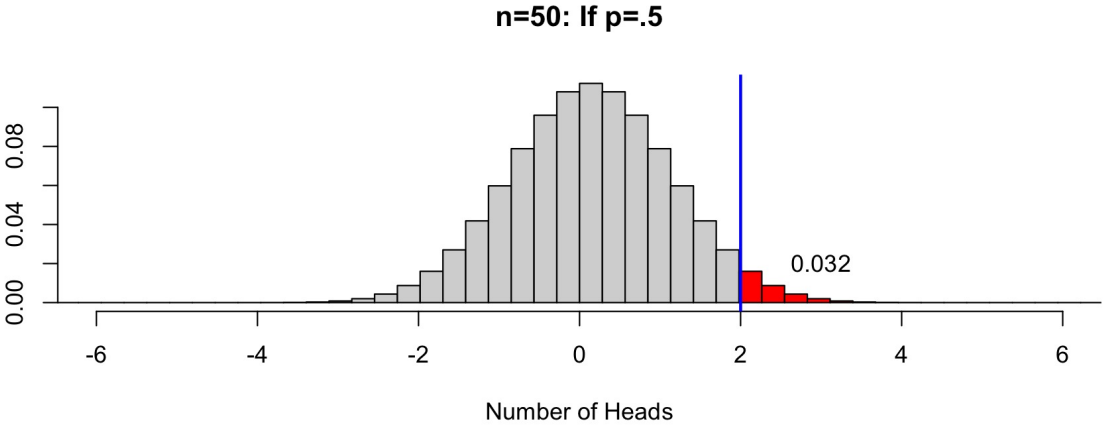
- What if the friend was cheating by different amount?

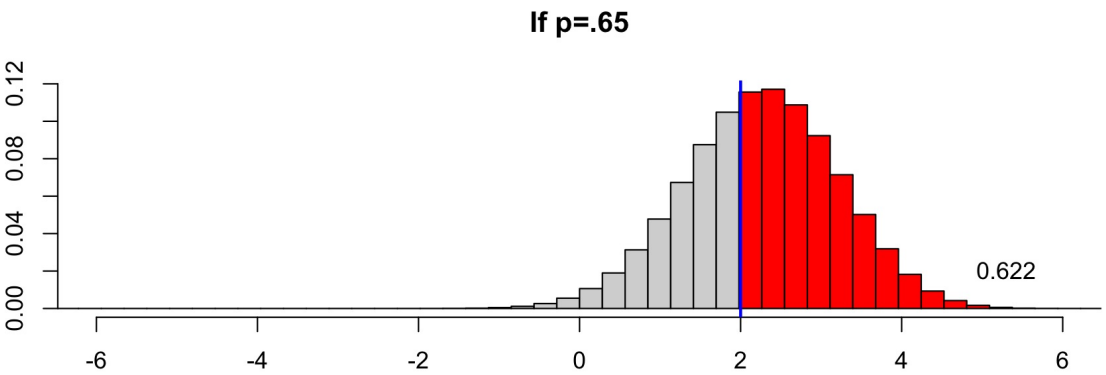

n=50: If p=.5
0.00
0.04
0.08
0.032
-6
-4
-2
0
2
4
6
Number of Heads


If p=.55
0.00
0.04
0.08
0.127
-6
-4
-2
0
2
4
6

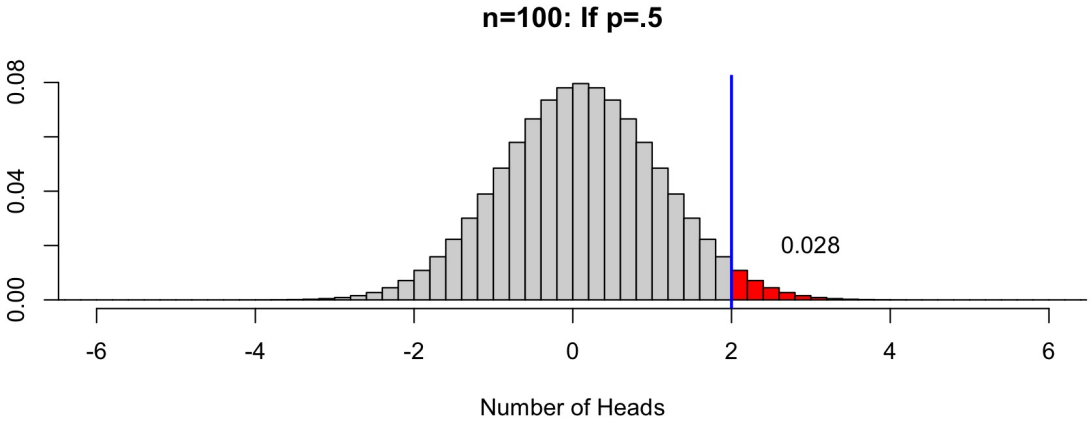

n=50: If p=.5
0.00
0.04
0.08
0.032
-6
-4
-2
0
2
4
6
Number of Heads

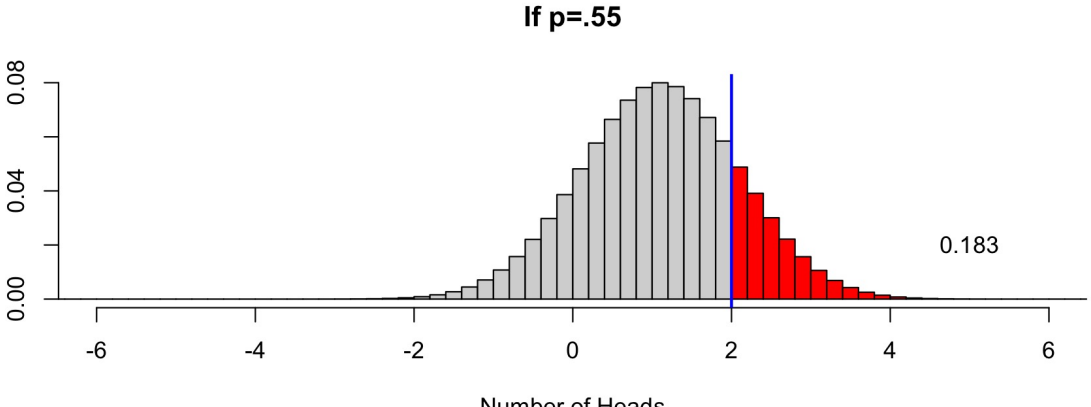

If p=.6
0.00
0.04
0.08
0.336
-6
-4
-2
0
2
4
6
Number of Heads

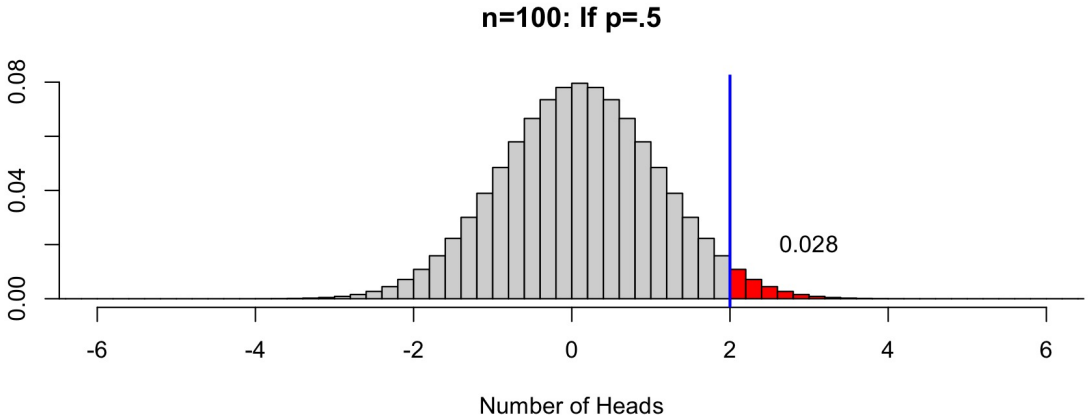

n=50: If p=.5
0.00
0.04
0.08
0.032
-6
-4
-2
0
2
4
6
Number of Heads

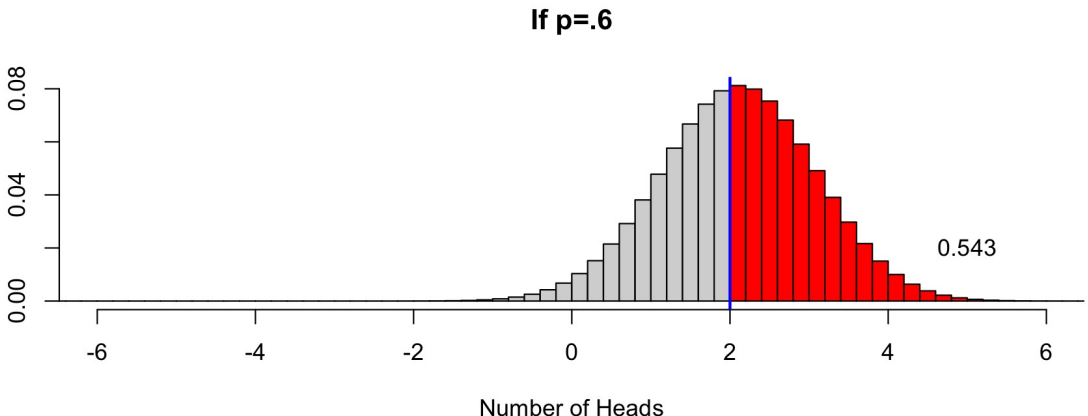

If p=.65
0.00
0.04
0.08
0.12
0.622
-6
-4
-2
0
2
4
6

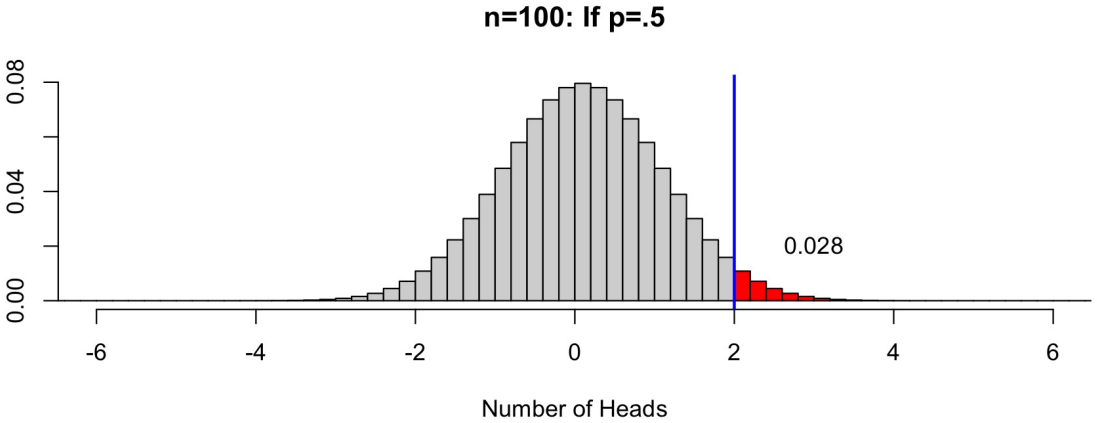
A.9 fig

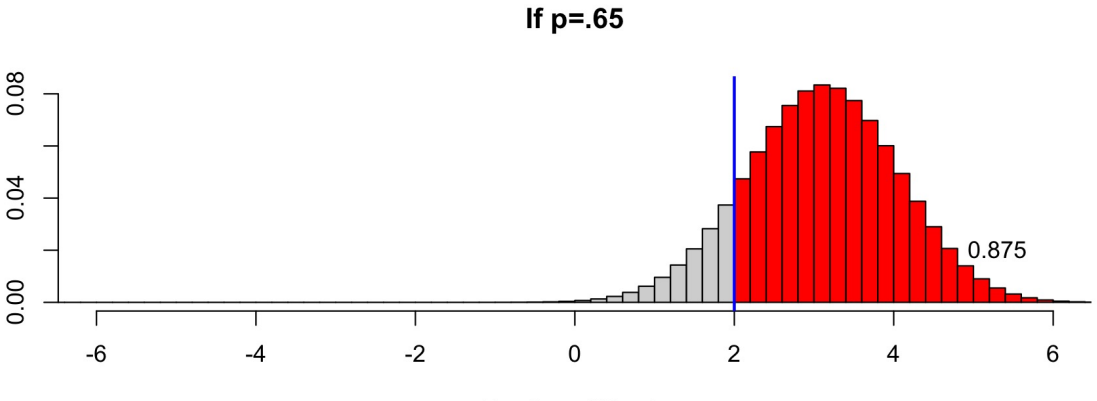

n=100: If p=.5
0.08
0.04
0.00
0.028
-6
-4
-2
0
2
4
6
Number of Heads


If p=.55
0.08
0.04
0.00
0.183
-6
-4
-2
0
2
4
6
Number of Heads


n=100: If p=.5
0.08
0.04
0.00
0.028
-6
-4
-2
0
2
4
6
Number of Heads


If p=.6
0.08
0.04
0.00
0.543
-6
-4
-2
0
2
4
6
Number of Heads


n=100: If p=.5
0.08
0.04
0.00
0.028
-6
-4
-2
0
2
4
6
Number of Heads


If p=.65
0.08
0.04
0.00
0.875
-6
-4
-2
0
2
4
6

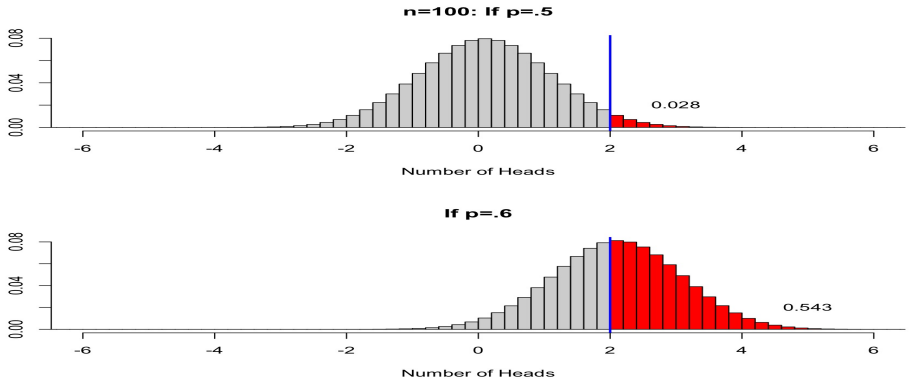
A.10 Type I and Type II Errors

- With significance level α of our choice,

Prob of ...	don't reject H_0	reject H_0
H_0 is true	$1-\alpha$	(α)
H_A is true	(β)	Power $= 1-\beta$

- $\alpha = P($ type I error $) = P($ false positive$)$
- $\beta = P($ type II error $) = P($ false negative$)$

A.11 P-value vs α



- p-value is the probability of getting the observed value of test statistics z or 'worse' when H_0 is true.

A.12 Test of Hypothesis - upper-tailed alternative

1. Set up the null and alternative hypothesis

$$H_0 : \qquad p = .5$$
$$H_A : \qquad p > .5$$

2. Calculate the test statistic

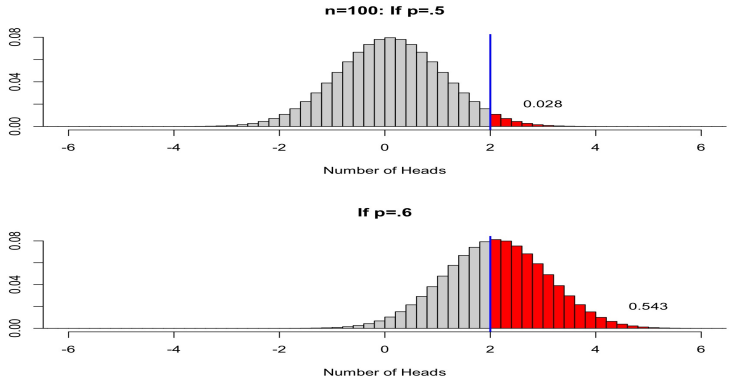
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

3. Get p-value

4. Conclude

If p-value is greater than α

If p-value is not greater than α



A.13 Test of Hypothesis - lower-tailed alternative

1. Set up the null and alternative hypothesis

$$H_0 : \qquad p = .5$$
$$H_A : \qquad p < .5$$

2. Calculate the test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

3. Get p-value

4. Conclude

If p-value is greater than α

If p-value is less than α

A.14 Test of Hypothesis - two-tailed alternative

1. Set up the null and alternative hypothesis

$$H_0 : \qquad p = .5$$
$$H_A : \qquad p \neq .5$$

2. Calculate the test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

3. Get p-value

4. Conclude

If p-value is greater than α

If p-value is less than α