8A One-Sample Z-test for Mean

Contents

Textbook: Devore 8e

8A Subsections

[ToC]

A.1 One-sample Z-test for μ

To test the null hypothesis of $H_0 : \mu = \mu_0$ against one of the alternatives from below:

$$
\begin{aligned}
H_A : &\quad \mu > \mu_0 \quad \text{(Upper-tailed alternative)} \\
H_A : &\quad \mu < \mu_0 \quad \text{(Lower-tailed alternative)} \\
H_A : &\quad \mu \neq \mu_0 \quad \text{(Two-tailed alternative)}
\end{aligned}
$$

We use the test statistic of

$$z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}},$$

and with significance level α,

$\mu_A = \frac{\mu - \mu_0}{\sigma/\sqrt{n}}$

Test procedure:

1. Set up the null and alternative hypothesis.

H_A	rejection region	p-value	Power
upper-tailed	$z > z_\alpha$	$1 - \Phi(z)$	1-$\Phi(z_\alpha - \mu_A)$
lower-tailed	$z < -z_\alpha$	$\Phi(z)$	$\Phi(-z_\alpha - \mu_A)$
Two-tailed	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2\big(1 - \Phi(\lvert z \rvert)\big)$	$1 - \Phi(z_{\frac{\alpha}{2}} - \mu_A) + \Phi(-z_{\frac{\alpha}{2}} - \mu_A)$

2. Calculate test statistic $z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$.

3. Calculate p-value according to the alternative.

4. Reject H_0 if p-value is LESS than α.
   If you can't reject H_0, then the test is inconclusive.

A.2 z-test vs t-test

1. When $n > 40$ $\rightarrow$ z-test
2. When $n \leq 40$ (Normality must be assumed)
   - σ is known $\rightarrow$ z-test
   - σ is unknown, s is used instead $\rightarrow$ t-test

A.3 Meaning of p-value

- p-value is the probability of getting the observed value of z or 'worse' when H_0 is true.