8C When N is less than 40

Contents

Textbook: Devore 8e

8C Subsections

[ToC]

C.1 Intro

1. When $n > 40$ $\rightarrow$ z-test
2. When $n \leq 40$ (Normality must be assumed)
   - σ is known $\rightarrow$ z-test
   - σ is unknown, s is used instead $\rightarrow$ t-test

C.2 One-sample t-test for μ

When we have to use S instead of σ

- Let $X_1, \ldots, X_n$ be Random Sample from $N(\mu, \sigma^2)$ distribution, and assume σ is **unknown**.
- We still have sample distribution,

$$\overline{X} \sim N(\mu, \sigma^2/n),$$

- But since σ is unkown, we must use the sample SD S instead.
- That gives us the test statistic

$$t = \frac{\overline{X} - \mu}{S/\sqrt{n}} \sim t(df = n - 1)$$

One Sample t-test

To test the null hypothesis of $H_0 : \mu = \mu_0$ against one of the alternatives from below:

$$H_A : \quad \mu > \mu_0 \quad \text{(Upper-tailed alternative)}$$
$$H_A : \quad \mu < \mu_0 \quad \text{(Lower-tailed alternative)}$$
$$H_A : \quad \mu \neq \mu_0 \quad \text{(Two-tailed alternative)}$$

We use the test statistic of $t = \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$,

H_A	rejection region	p-value	power
upper-tailed	$t > t_{\alpha,n-1}$	$1 - P(T < t)$	$1 - P(T < t_\alpha - \mu_A)$
lower-tailed	$t < -t_{\alpha,n-1}$	$P(T < t)$	$P(T < -t_\alpha - \mu_A)$
Two-tailed	$t < -t_{\alpha/2,n-1}$ or $t > t_{\alpha/2,n-1}$	$2\big(1 - P(T < \lvert t \rvert)\big)$	$1 - P(T < t_{\frac{\alpha}{2},n-1} - \mu_A)$ $+P(T < -t_{\frac{\alpha}{2},n-1} - \mu_A)$

$$T \sim t(n-1), \qquad \mu_A = \frac{\sqrt{n}}{\sigma}(\mu - \mu_0)$$

C.3 Ex: Concrete Strength

[ToC]

- Standards for 28-day compressive strength test for concrete requies that concrete cylinders have true mean compressive strength higher than 3000 psi (20.7 MPa).
- 15 specimen has been tested, and resulted in sample mean of 3055 psi, and sample standard deviation of 75 psi.
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n=15 xbar=3055 S = 75 mu0=3000 t = (xbar - mu0)/(S/sqrt(n)) 1-pt(t, n-1) pt(t, n-1) 0.006550495

C.4 Ex: Sprinkler Systems

- Manufacturer claims true average system temparature is 130 F. Sample of n=9 was taken with sample average of 131.08 F, and sample standard deviation of 1.5F. Test the hypothesis that manufacturer is right against upper-tail alternative.

n=9 xbar=131.08 mu0=130 S = 1.5 t = (xbar - mu0)/(S/sqrt(n)) 1-pt(t, n-1) pt(t, n-1)
1] 0.0313947

C.5 Ex: Tire Life

- The manufacturer of a new fiberglass tire claims that its average life will be at least 40,000 miles. To verify this claim a sample of 12 tires is tested. Sample mean was 37.84, and sample SD was 2.56.

n=12 xbar = 38.84 S = 3.56 mu0 = 40 t = (xbar - mu0)/(S/sqrt(n)) 1-pt(t, n-1) pt(t, n-1) 0.1415091