9B Power Analysis

Contents

Textbook: Devore 8e

9B Subsections

[ToC]

B.1 Ex: Hydro Turbines

- New model tested: n=10, sample mean 27.1, sigma is known to be 5.2.
- Is this an evidence that μ is higher than 25.2 (old model)?

```
mu0=25.2;    Xbar=27.1;    Si=5.2;    n=10
z = (Xbar-mu0) / (Si/sqrt(n))
z
1.16

upper-tailed alt: pval = 1-pnorm(1.16)
0.123
```

B.2 Two Questions

- We failed to reject H_0, Do we have hope? Should we continue testing?
- Did the test come out as inconclusive, because μ is so close to 25.2?
- Or because the sample size was not enough?

State your worst-case acceptable, and check the power.

Suppose $\mu = 26.2$ is good enough for new model (if we can show evidence for it).

B.3 aaa

```
For mu=26.2
mu_A = (mu-25.2) / (5.2/sqrt(10));  mu_A=0.6081
Power = 1-pnorm(1.65-mu_A);         Power=0.1487

# If you increase to
n=60
mu_A = (mu-25.2) / (5.2/sqrt(60));  mu_A=1.49
Power = 1-pnorm(1.65-mu_A);         Power=0.4363
```

Suppose μ was actually as good as 27.0.

```
mu=27.0
mu_A = (mu-mu0)/Si*sqrt(n); mu_A          # mu_A =1.1
power = 1-pnorm(1.65, mu_A, 1); power     # power=.2893

n=40
mu=27.0
mu_A = (mu-mu0)/Si*sqrt(n); mu_A          # mu_A =2.19
power = 1-pnorm(1.65, mu_A, 1); power     # power=.7051

n=80
mu=27.0
mu_A = (mu-mu0)/Si*sqrt(n); mu_A          # mu_A =3.09
power = 1-pnorm(1.65, mu_A, 1); power     # power=.9259
```

B.4 Formula for Power

One-sample Z-test for μ To test the null hypothesis of $H_0 : \mu = \mu_0$ against one of the alternatives from below:

$$H_A : \quad \mu > \mu_0 \quad \text{(Upper-tailed alternative)}$$
$$H_A : \quad \mu < \mu_0 \quad \text{(Lower-tailed alternative)}$$
$$H_A : \quad \mu \neq \mu_0 \quad \text{(Two-tailed alternative)}$$

With significance level α, we use the test statistic of $z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$.

$$\mu_A = \frac{\mu - \mu_0}{\sigma/\sqrt{n}}$$

H_A	rejection region	p-value	Power
upper-tailed	$z > z_\alpha$	$1 - \Phi(z)$	1-$\Phi(z_\alpha - \mu_A)$
lower-tailed	$z < -z_\alpha$	$\Phi(z)$	$\Phi(-z_\alpha - \mu_A)$
Two-tailed	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2\big(1 - \Phi(\lvert z \rvert)\big)$	$1 - \Phi(z_{\frac{\alpha}{2}} - \mu_A) + \Phi(-z_{\frac{\alpha}{2}} - \mu_A)$

B.5 When you want to 'accept' the null

- P-value is less than $\alpha \rightarrow$ Reject H_0 (Conclusive. Evidence toward H_A.)
- P-value is greater than $\alpha \rightarrow$ Can't reject H_0 (Inconclusive. No evidence toward H_A.)

B.6 Sometimes, we wish to 'accept' H_0.

- We want to show that μ is probably not too far from μ_0.
- We cann't accept H_0 just because we could not reject H_0.
- We must make sure the power is high for the 'worst case acceptable'.

B.7 Ex: Lab Scale

- To assess the accuracy of a laboratory scale, a standard weight that is known to weigh exactly 1 gram is repeatedly weighed a total of 25 times.
- $\bar{X}$ is computed to be 1.0028 grams.
- Suppose the each scale reading is independent of each other, and Normally distributed with unknown mean μ and standard deviation $\sigma = .01g$.
- μ is supposed to be very close to 1 (within $.001g$.)

95% CI = Xbar pm .00392 = (.99888, 1.00672)
Test if mu=1. z=1.4 p-val = .161

Power of this test if mu=1.001 Pow=.125

Get n that will give margin of error = $.001$. (n=385)

What is the prob that \mu is within the interval ($\bar X-.001, \bar X+.002$)?

B.8 Ex: pH meter bias

Suppose that an engineer is interested in testing the bias in a pH meter. Data are collected on a neutral substance (pH=7.0). A sample of the measurements were taken with the data as follows:

```
x <- c(7.07, 7.00 , 7.10 , 6.97 , 7.00 , 7.03 , 7.01, 7.01, 6.98, 7.08)
(mean(x) - 7)/ (sd(x)/sqrt(10))

1-pnorm(1.96 - (7.01 - 7)/ (sd(x)/sqrt(10)) )
```

B.9 Ex: Prescription

- A certain prescription medicine is supposed to contain an average of 247 parts per million(ppm) of a certain chemical.
- n=20, sample mean = 250ppm, sample SD 12ppm.
- Test against two sided alternative. Power and P(type II) when true mean is 253?

```
mu0=25.2;    Xbar=27.1;    Si = 5.2;    n=10

z = (Xbar-mu0) / (Si/sqrt(n)) ; z       # z=1.16

#--- Upper-tail Alternative ---
Pval = 1-pnorm(z); Pval                 # pval=0.1240.
```