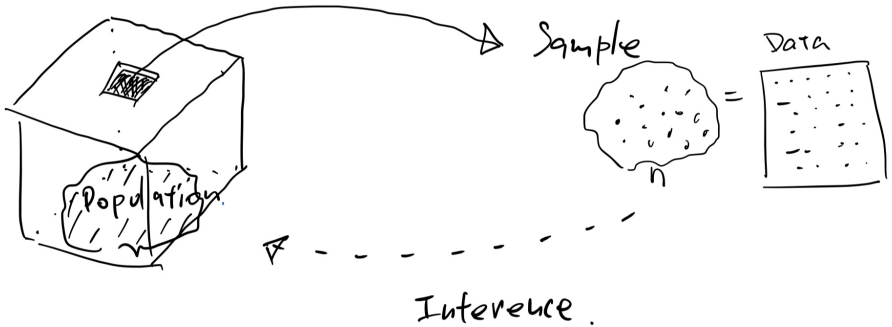
2A Prob Basics

Contents

2A Probability Basics

[ToC]

A.1 Probability and Statistics



A.2 What does Prob = 1/6 mean?

Example: Roll a die:

$$P\Big(\text{get } \#\ 3\Big) =$$

- What does this mean?
- When do we actually see $\frac{1}{6}$ in our real-life dice-rolling?

A.3 It means Rel Freq converges to 1/6 if n is large

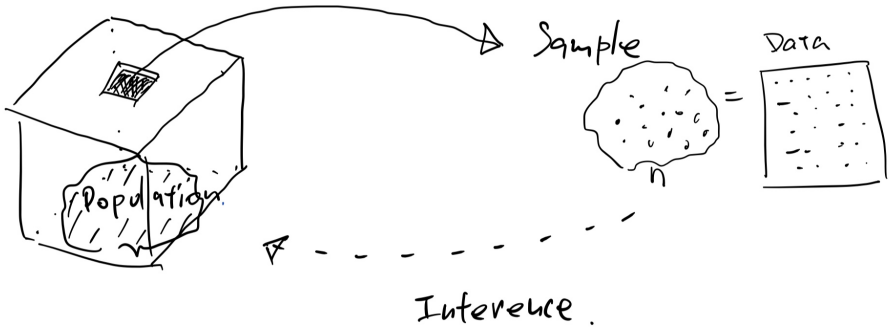
Interpretation of Probability

- Relative frequeny gets closer and closer to probability as number of trial increases.

$$[\text{Relative Frequency}] \quad \Rightarrow \quad [\text{Probability}] \quad \text{as } n \to \infty \, .$$

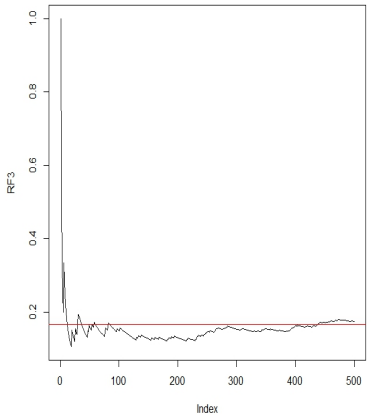
$$\frac{\{\text{num of times the die shows 3}\}}{\{\text{number of rolls}\}} \quad \Rightarrow \quad [\text{Probability}]$$

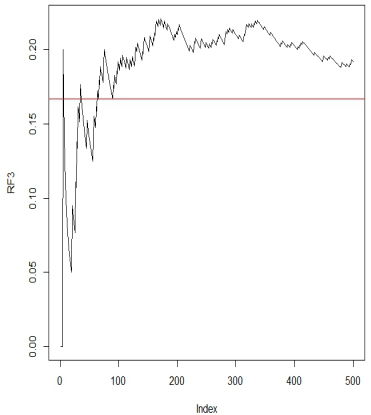
A.4 Statistics and Probability



A.5 We see prob only when n is 'large'

- Frequentist interpretation of Probability
- Sample plot from simulation


RF3
Index
0
100
200
300
400
500
0.2
0.4
0.6
0.8
1.0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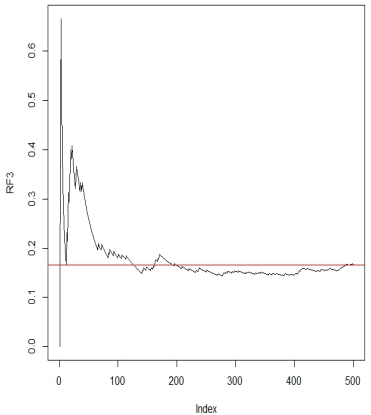
RF3
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0.0
0.1
0.2
0.3
0.4
0.5
0.6

A.6 Sample Space and Events

- **Experiment** is any action or process whose outcome is subject to uncertainty.
  (e.g. roll a die)
- **Sample Space** of an experiment is **a set of all possible outcomes**.
  (e.g. $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$)
- **Event** is any subset of the sample space $\mathcal{S}$.
  (e.g. {1,2,3})

A.7 Ex: Roll a die

- sample space $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$.
- an event A =(number less than 4) =
- an event B =(number is odd) =
- What is

$$P(A) = ?$$

A.8 When each outcome is equally likely

If each element in $\mathcal{S}$ is equally likely, then for any event A,

$$P(A) = \frac{[\text{number of elements in } A]}{[\text{number of elements in } \mathcal{S}]}$$

A.9 Ex: Flip a coin twice

Let X =(number times you get head).

- What is $\mathcal{S}$?
- What is $P(X = 2)$?
- There are two ways to write $\mathcal{S}$.

A.10 Ex: Kids Next Door

- Two kids moved in next door
- You saw one of them was a girl
- P(The other one is also a girl)?

A.11 Ex: Sum of two dice

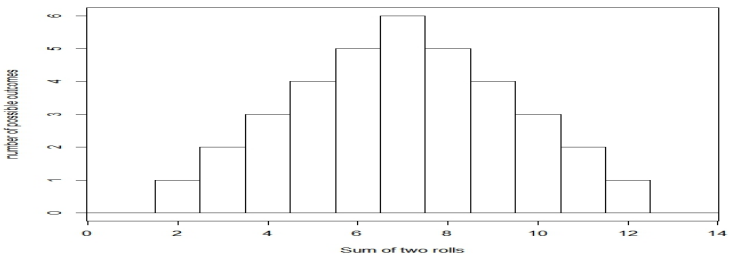
Roll two dice, then sum the two number. Let X be the sum.

- How do you write $\mathcal{S}$?
- What is the most likely outcome ?

Write $\mathcal{S}$ in a form (First Throw, Second Throw):

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	**(1,6)**
(2,1)	(2,2)	(2,3)	(2,4)	**(2,5)**	(2,6)
(3,1)	(3,2)	(3,3)	**(3,4)**	(3,5)	(3,6)
(4,1)	(4,2)	**(4,3)**	(4,4)	(4,5)	(4,6)
(5,1)	**(5,2)**	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)



A.12 What if X =minimum of two numbers

What is

$$P(X > 4) = ?$$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)