2B Counting Techs

Contents

2B Counting Techniques

[ToC]

B.1 Counting Formulas

Select k out of n:

	without replacement	with replacement
ordered	$\frac{n!}{(n-k)!} = {}_nP_k$	n^k
not ordered	$\binom{n}{k} = {}_nC_k$	$\binom{n+k-1}{k}$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

B.2 Ex: Orderd, without Replacement

Example: If you have 6 cards labeled A, B, C, D, E, F, how many different sequences can you make?

Example: If you have 6 cards labeled A, B, C, D, E, F, how many different sequences can you make with only using 4 cards?

B.3 Counting Formula 1 (Permutations)

- When you have n subjects, there are $n!$ ways to order.
- When you have k subjects out of n subjects, there are $n!/(n-k)!$ ways to order.

B.4 Counting Formulas

Example: If you have 6 cards labeled A, B, C, D, E, F, how many different groups can you make with 3 cards?

B.5 Counting Formula 2 (Not ordered, without Replacement)

Example: If you have 6 cards labeled A, B, C, D, E, F, how many different groups can you make with 3 cards?

- When you choose k subjects out of n, without regard to order, there are

  $$\binom{n}{k} = \frac{n!}{(n-k)!\ \ k!}$$

  possible combinations.

- This is read as "n choose k".

- Some calculater write this as ${}_nC_k$

B.6 Binomial Coefficient

- Binomial Coefficient:

$$\binom{n}{k} = \frac{n!}{(n-k)!\ \ k!}$$

- Binomial Expansion:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

- Binomial Tree

- Can you expand $(x+y)^7$?

B.7 Counting Formulas

Select k out of n:

	without replacement	with replacement
ordered	$\frac{n!}{(n-k)!} = {}_nP_k$	n^k
not ordered	$\binom{n}{k} = {}_nC_k$	$\binom{n+k-1}{k}$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

B.8 Ex: Batting Orders

1. There are 9 players in a baseball team. How many different batting orders are possible?

2. What if you have 15 players ? (only 9 can play)

3. What if there are 3 pichers(have to bat 9th) 5 sluggers (have to bat clean up (3rd, 4th, 5th) and 7 players?

B.9 Ex: 20 people in a party

If everybody shakes hand with everybody, how many handshakes occur?

B.10 Ex: Binomial Expansion

When you expand $(2x^2 + y)^5$, what is the coefficient for term with x^6y^2?

B.11 Ex: Kids and Gifts

Seven different gifts are distributed among 10 kids. One kid can't get more than 1 gift. How many different ways?

What if the gifts are identical?

B.12 Ex: Cards

How many different sequence can you make with cards $AAABBCCCC$?

B.13 Counting Formula 3 (Ordered, with Replacement)

Example: PIN number is made of 4 digit number of 0-9. How may possible PINs are there?

Example: Password for a website must be 6 characters long, and for each character, you can use any of alphabet and number. How many different password can you make?

B.14 Counting Formula 4 (Not Ordered, with Replacement)

There are 7 spices A,B,C,D,E,F,G. Suppose you randomly pick 5 pinches of spice, out of 7 with with replacement. How may different combinations can you make?

This is then same as having 11 slots and choose 5 for stars, or have 11 slots and choose 6 for dividers.

5 stars, 7-1 dividers

$* \quad * \quad * \quad * \quad *$

$* \quad * \quad * \quad * \quad *$

$* \quad * \quad * \quad * \quad *$

$* \quad * \quad * \quad * \quad *$

B.15 Counting Formulas

Select k out of n:

	without replacement	with replacement
ordered	$\frac{n!}{(n-k)!} = {}_nP_k$	n^k
not ordered	$\binom{n}{k} = {}_nC_k$	$\binom{n+k-1}{k}$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$