2D Prob through Set Op

Contents

2D Probability through Set Operations

[ToC]

D.1 Set Operations

$$\mathcal{S} = \{1, 2, 3, 4, 5, 6, 7, 8\},$$
$$A = \{1, 3, 5, 7\}, \qquad B = \{2, 4, 5\} \qquad D = \{6, 8\}$$

then

$$\text{Union: } A \cup B = \{1, 2, 3, 4, 5, 7\}$$
$$\text{Intersection: } A \cap B = \{5\}$$
$$\text{Complement: } B^c = \{1, 3, 6, 7, 8\}$$
$$\text{Disjoint if } A \cap D = \{\varnothing\}$$
$$\text{Exhaustive if } A \cup B \cup D = \mathcal{S}$$

D.2 Distributive law of unions and intersections

Unions and intersections can be distributed:

$$A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$$
$$A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$$

D.3 DeMorgan's law

Distributive law of complement over union or intersection

$$(C_1 \cap C_2)^c = C_1^c \cup C_2^c$$
$$(C_1 \cup C_2)^c = C_1^c \cap C_2^c$$

$$(C_1 \cap C_2 \cap C_3)^c = C_1^c \cup C_2^c \cup C_3^c$$
$$(C_1 \cup C_2 \cup C_3)^c = C_1^c \cap C_2^c \cap C_3^c$$

$$\left(\cap_{k=1}^{\infty} C_k \right)^c = \cup_{k=1}^{\infty} C_k^c$$
$$\left(\cup_{k=1}^{\infty} C_k \right)^c = \cap_{k=1}^{\infty} C_k^c$$

D.4 Axioms of probability

P is a probability set function if

1. $P(A) \geq 0$, for all event $A \in \mathcal{B}$.
2. $P(\mathcal{S}) = 1$.
3. If C_n is a sequence of disjoint events in $\mathcal{B}$, then

$$P\Big(\cup_{n=1}^{\infty} C_n \Big) = \sum_{n=1}^{\infty} P(C_n)$$

D.5 Probability Formulas

1. $P(A^c) = 1 - P(A)$
2. $P(A) = P(A \cap B) + P(A \cap B^c)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Inclusion-Exclusion (Can be extended)
4. $(A \cap B)^c = A^c \cup B^c$ DeMorgan's law
5. $(A \cup B)^c = A^c \cap B^c$

D.6 Ex: Project Funding

There are 3 projects that has applied for the grant. Let A_i represent an event that project i gets funded. There are 3 projects. Given

$$P(A_1) = .22, \quad P(A_2) = .25, \quad P(A_3) = .28$$

and

$$P(A_1 \cap A_2) = .11, \qquad P(A_1 \cap A_3) = .05,$$
$$P(A_2 \cap A_3) = .07, \qquad P(A_1 \cap A_2 \cap A_3) = 0.01,$$

Calculate the probability of :

1. P(At least one of project 1 and 2 get award)
2. P(Neither project 1 nor 2 get award)
3. P(Only project 3 is awarded)

D.7 Ex: Lab (work vs Referral)

D.8 Inc-Exc Extended