2G LoTP and Bayes'

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2G The Law of Total Probability and Bayes' Theorem

[ToC]

G.1 The Law of Total Probability

- Recall formula:

$$P(B \cap A) \quad = P(B|A)P(A).$$

  Then for event B, can be written using formula #2,

$$P(B) \qquad = P(B \cap A) + P(B \cap A')$$

$$= P(B|A)P(A) + P(B|A')P(A')$$

- Instead of A, A', if A_1, A_2, A_3 are mutually exclusive and exhaustive events, we can write

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

G.2 Ex: Balls in Two Urns

- There are two urns, urn A and urn B.
- Urn A contains 5 red balls, 2 white.
- Urn B contains 3 red balls, 4 white.
- Fair coin flip decides which urn to be used. What is the probability that Red will be picked?

G.3 Ex: Balls in Two Urns 2

- Urn A contains 5 red balls, 2 white.
- Urn B contains 3 red balls, 4 white.
- Given that the ball picked was Red, what is the probability urn A was used?

G.4 Bayes' Theorem

- Bayes' formula says:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

G.5 Ex: Testing for Disease

- 1 in 1000 adults is afflicted with this disease.
- Test for this disease is 99% accurate on infected patients.
- Test is 98% accurate on non-infected patients.
- If test comes back positive, what is the chance that you are actually infected?

$P\big(\text{Infected}|\text{Pos}\big) =$

$P\big(\text{Not Infected}|\text{Neg}\big) =$

G.6 Ex: Testing Positive Twice

What if the person who tested positive once, takes the same test, and test positive again? Now what is the probability that the patient is infected?

G.7 Ex: Driver Class

Suppose all drivers are divided into three categories A,B, and C. It is known that among all drivers, 50% of them are class A driver, 30% are class B driver, and 20% are class C driver. Probabilities for numbers of accidents each driver have in a year are given below:

	0	1	2	3+
A	.7	.15	.1	.05
B	.5	.25	.15	.1
C	.3	.3	.25	.15

A driver had one accident last year. Given this information, what is the probability he is an class A driver? How about probability that he is an class B driver? Class C driver?