3C-2 MGF

Contents

3I Moment Generating Function

[ToC]

- MGF for r.v. X is defined as

$$M_X(t) = E(e^{tX})$$

- Generates moments $(E[X^k])$ as its derivatives at $t = 0$.

$$\begin{aligned} M'_X(0) &= E(X) \\ M''_X(0) &= E(X^2) \\ M'''_X(0) &= E(X^3) \\ &\vdots \end{aligned}$$

I.1 Notes

- It may not exist for some r.v.
- mgf only need to exist in an open neighbourhood around 0.
- If they do exist, then mgf specifies cdf.
- Characteristic Function $E(e^{itX})$ always exsits, and specifies cdf.
- Very useful in showing the distribution for sum of independent r.v.
  If $Y = X_1 + X_2$, then

  $$M_Y(t) = E\left[e^{tY}\right] = E\left[e^{t(X_1+X_2)}\right] = E\left[e^{t(X_1)}e^{t(X_2)}\right]$$

  If X_1 and X_2 are independent, then

  $$M_Y(t) = E\left[e^{t(X_1)}\right]E\left[e^{t(X_2)}\right] = M_{X_1}(t)M_{X_2}(t)$$

I.2 Example:

- Suppose r.v. X has mgf $M_X(t) = 1/(1-t)$. Calculate its mean and variance.
- Obtain pdf of X.

I.3 Ex: MGF of Binary RV

Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} 1/3 & \text{for } x = 0 \\ 2/3 & \text{for } x = 1 \\ & \text{otherwise} \end{cases}$$

Determine the moment generating function of $Y = X_1 + X_2 + X_3$.