3C Variance

Contents

3C Variance (Theoretical)

[ToC]

C.1 Variance

- Variance of a random variable X, $V(X)$, is defined as

$$\sigma^2 = E\Big[\big(X-\mu\big)^2\Big] \quad = \quad \sum_{i=1}^{n}(x_i-\mu)^2 \cdot p(x_i)$$

  Note that $\mu = E(X)$. Standard Deviation of X is defined as $\sigma = \sqrt{\sigma^2}$.

- Compare to Sample Variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

C.2 Alternative formula for variance

$$
\begin{aligned}
V(X) &= E\Big[(x-\mu)^2\Big] \\
&= E\Big[x^2 - 2\mu x + \mu^2\Big] \\
&= E[x^2] + E[-2\mu x] + E[\mu^2] \qquad E \text{ can be distributed over any linear equation.}
\end{aligned}
$$

But $\mu = E(X)$, and they are just a number. Taking E again doesn't do anything. So,

$$
\begin{aligned}
V(x) &= E[x^2] - 2\mu E[x] + \mu^2 \\
&= E[x^2] - 2\mu^2 + \mu^2 \\
&= E[x^2] - \mu^2
\end{aligned}
$$

Similarly to the discrete case, we have

$$V(aX+b) = a^2 V(X)$$

C.3 Ex: Throw a Die Once

Suppose you are given pmf:

#	1	2	3	4	5	6
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- Expectation was $E(X) = 3.5$.
- Now calcualte $E(X^2)$,

$$E(X^2) \quad = 1^2 \cdot \tfrac{1}{6} + 2^2 \cdot \tfrac{1}{6} + 3^2 \cdot \tfrac{1}{6} + 4^2 \cdot \tfrac{1}{6} + 5^2 \cdot \tfrac{1}{6} + 6^2 \cdot \tfrac{1}{6} = 15.167$$

- So the variance is

$$V(X) = E(X^2) - [E(X)]^2 = 15.167 - 3.5^2 = 2.917.$$

Example: Casino Simplified

- .995 chance for $1 profit.
- .005 chance for $-99 profit (loss).
- Calculate Variance of game profit.