3F Negative Binomial

Contents

3F Negative Binomial Distributions

[ToC]

F.1 Negative Binomial (Flips ver. as in Wackerly)

$X \sim NegBin(r, p)$ **Analogy:** Number of **flips** until you get r heads.

$$\text{pmf}: \quad p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \quad \text{for } x = r, r+1, r+2, \ldots$$

$$\text{CDF}: \quad F(x) = P(X \leq x) = \sum_{k=0}^{x} p(x)$$

$$\text{mean}: \quad E(X) = \frac{r}{p}$$

$$\text{var}: \quad V(X) = \frac{r(1-p)}{p^2}$$

$$\text{MGF}: \quad M(t) = [\frac{pe^t}{1-(1-p)e^t}]^r$$

Called Geometric Distribution if $r = 1$.

```
dnbinom( 10, r,  p)   # pmf at x=10 flips (In R, X=# of flips)
pnbinom( 10, r,  p)   # CDF at x=10 flips
pnbinom(.5,  r,  p)   # Inv CDF at q=.5
rnbinom(1000, r, p)   # random sample of size 1000
```

F.2 Negative Binomial (Tails ver. as in Devore)

$X \sim NegBin(r, p)$ **Analogy:** Number of **TAILS** until you get r heads.

$$\text{pmf}: \quad p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x \quad \text{for } x = r, r+1, r+2, \ldots$$

$$\text{CDF}: \quad F(x) = P(X \le x) = \sum_{k=0}^{x} p(x)$$

$$\text{mean}: \quad E(X) = \frac{r(1-p)}{p}$$

$$\text{var}: \quad V(X) = \frac{r(1-p)}{p^2}$$

$$\text{MGF}: \quad M(t) = [\frac{pe^t}{1-(1-p)e^t}]^r$$

Called Geometric Distribution if $r = 1$.

```
dnbinom( 10-r, r,  p)   # pmf at x=10 tails  (In R, X=# of flips)
pnbinom( 10-r, r,  p)   # CDF at x=10 tails
pnbinom(.5,  r,  p)   # Inv CDF at q=.5
rnbinom(1000, r, p)   # random sample of size 1000
```

F.3 NB on R

$X \sim \text{NB}(r = 5, p = .4)$ [X=num of Failures]

```
X = rnbinom(1000, 5, .4)
plot(X)
hist(X)

dnbinom(x=2, 5, .4)
pnbinom(x=2, 5, .4)

t = seq(0,10)
plot( t, dnbinom(t,5,.4) )
plot( t, pnbinom(t,5,.4),type='s' )
```

F.4 Pmf and CDF

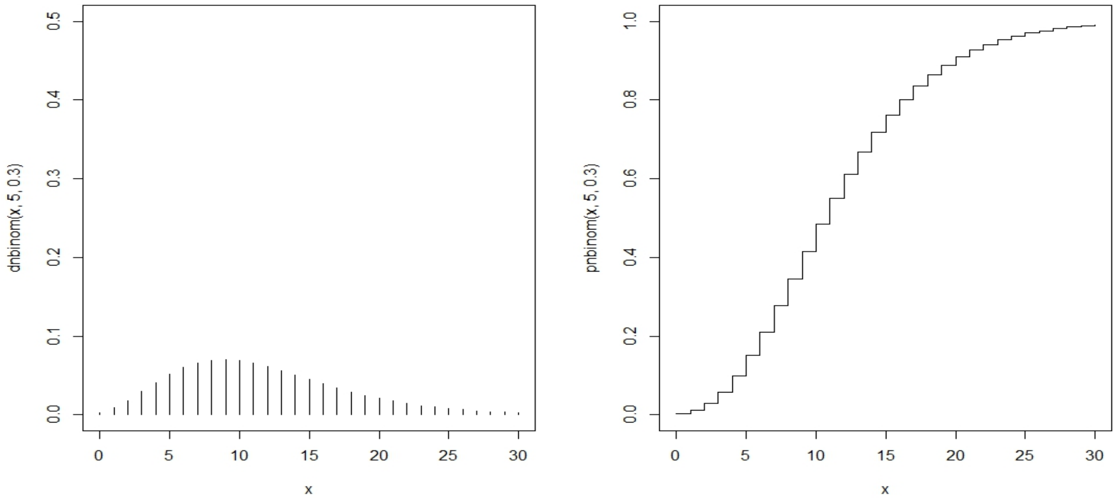

Figure 1: r=5, p=.3

F.5 Ex: Shoot free throw until you make 10 shots

Suppose your free-throw percentage is 90%. Assume independence between each shots. You can't go home until you make 10 baskets, how many shots do you need to take before you go home?

F.6 Ex: Win before You Lose

- One concern of a gambler is that she will go broke before achieving her first win.
- Suppose that she plays a game in which the probability of winning is .1 (and is unknown to her).
- It costs her \$10 to play and she receives \$80 for a win.
- If she commences with \$30, what is the probability that she wins exactly once before she loses her initial capital?