Ch8 - Decision Tree

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Textbook: James et al. ISLR 2ed.

8A Subsection

[ToC]

A.1 Basics

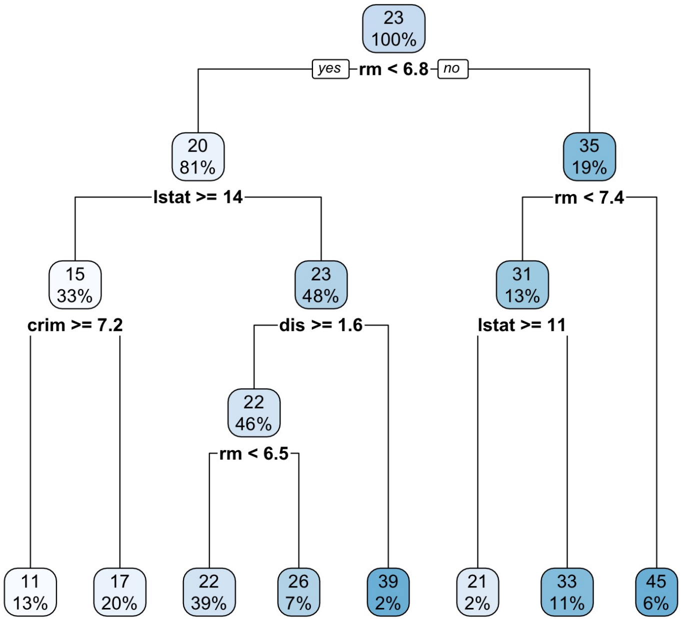
Boston Data (400 training, 96 test)

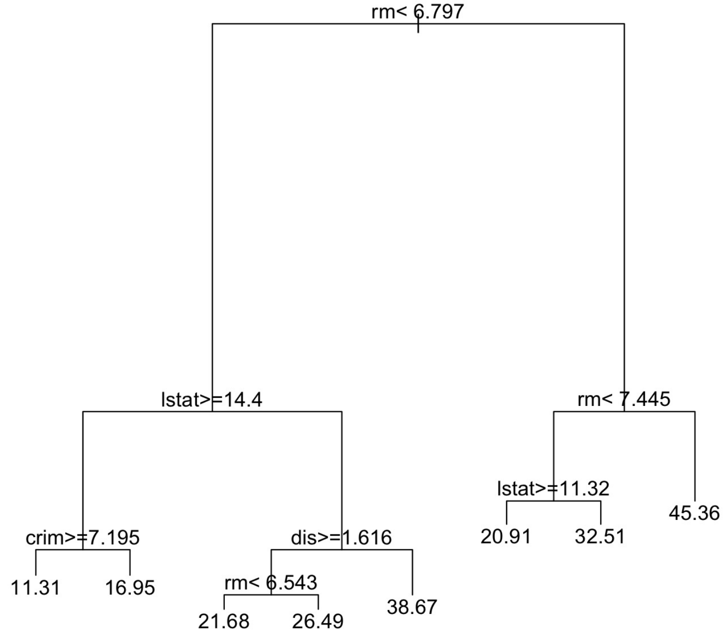
```
Reg01 = lm(medv~., data=Train.set)
> summary(Reg01)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  35.023724   6.198554   5.650 3.11e-08 ***
crim         -0.115721   0.037334  -3.100 0.002080 **
zn            0.041696   0.016893   2.468 0.014013 *
indus        -0.013844   0.072338  -0.191 0.848329
chas          2.666581   1.005233   2.653 0.008315 **
nox         -16.880242   4.557814  -3.704 0.000244 ***
rm            3.928494   0.498349   7.883 3.30e-14 ***
age          -0.002590   0.015995  -0.162 0.871431
```

```
dis          -1.514405   0.237707  -6.371 5.35e-10 ***
rad           0.296458   0.075489   3.927 0.000102 ***
tax          -0.010113   0.004365  -2.317 0.021054 *
ptratio      -0.925880   0.150607  -6.148 1.96e-09 ***
black         0.008767   0.003380   2.594 0.009849 **
lstat        -0.520376   0.060880  -8.548 2.97e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.958 on 386 degrees of freedom
Multiple R-squared:  0.7303,Adjusted R-squared:  0.7212
F-statistic: 80.41 on 13 and 386 DF,  p-value: < 2.2e-16
```


23
100%
yes
rm < 6.8
no
20
81%
lstat >= 14
35
19%
rm < 7.4
15
33%
crim >= 7.2
23
48%
dis >= 1.6
31
13%
lstat >= 11
22
46%
rm < 6.5
11
13%
17
20%
22
39%
26
7%
39
2%
21
2%
33
11%
45
6%



rm< 6.797
lstat>=14.4
rm< 7.445
lstat>=11.32
45.36
crim>=7.195
dis>=1.616
20.91
32.51
11.31
16.95
rm< 6.543
38.67
21.68
26.49

A.2 Decision Tree

- Can be used in Classification / Regression
- Root Node / Parent Node / Child Node
- Terminal Nodes / Leaves
- Every observation in the terminal node gets same prediction (mean of region)
- Regression Tree: minimize RSS

$$\sum_{j=1}^{J} \sum_{i \in R_j} (y_i - \hat{y}_{R_j})^2$$

- Classification Tree: 3 choices for the measure.

- Classification Error Rate
- Gini Index
- Entropy (Cross-Entropy) Gini and Entropy are numerically similar.

- Recursive Binary Splitting: Can't go through all the possibilities; Use Top-Down approach.

A.3 Tree Growing

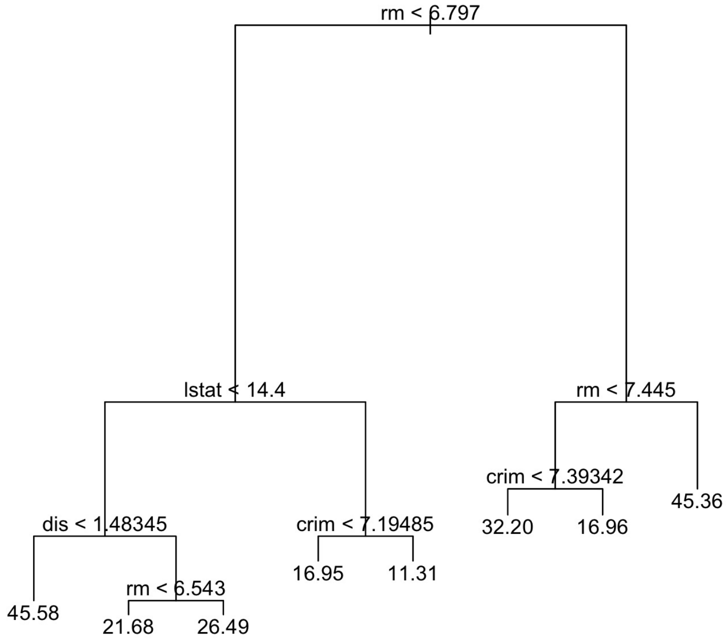
Recursive Binary Splitting

1. Pick the 1st feature, and go through all possible splitting points s, calculating (RSS/Entropy) at each point.

2. Go throgh all featurs, pick the feature that gives min (RSS/Entropy). That's the best feature to be split at.

3. Repeat. (feature that was used for previous split is still in the pool)

```
#1  crim     per capita crime rate by town.
#2  zn       proportion of residential land zoned for lots over 25,000 sq.ft.
#3  indus    proportion of non-retail business acres per town.
#4  chas     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise).
#5  nox      nitrogen oxides concentration (parts per 10 million).
#6  rm       average number of rooms per dwelling.
#7  age      proportion of owner-occupied units built prior to 1940.
#8  dis      weighted mean of distances to five Boston employment centres.
#9  rad      index of accessibility to radial highways.
#10 tax      full-value property-tax rate per $10,000.
#11 ptratio  pupil-teacher ratio by town.
#12 black    1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town.
#13 lstat    lower status of the population (percent).
#14 medv     median value of owner-occupied homes in \$1000s.
```


rm < 6.797
lstat < 14.4
rm < 7.445
crim < 7.39342
45.36
dis < 1.48345
crim < 7.19485
32.20
16.96
16.95
11.31
rm < 6.543
45.58
21.68
26.49

A.4 Tree Pruning

- Seemingly worthless split may lead to large reduction in RSS later on. (Grow more)
- Growing is likely to overfit, because the resulting tree might be too complex. (Grow less)

- A smaller tree with fewer splits might lead to lower variance and better interpretation at the cost of a little bias.
- Grow a large tree (stop only when each terminal node has fewer than some min num of obs. Then prune later.

A.5 Pruning

- Grow a large tree, then try to select a subtree that leads to the lowest TEST error rate.
- Given a subtree, we can estimate its test error using CV.
- Since there can be too many subtrees, cost complexity pruning selects a small set of subtrees for consideration. (AKA weakest link pruning).
- Rather than considering every possible subtree, we consider a sequence of trees indexed by a nonnegative tuning parameter α.

A.6 Pruning

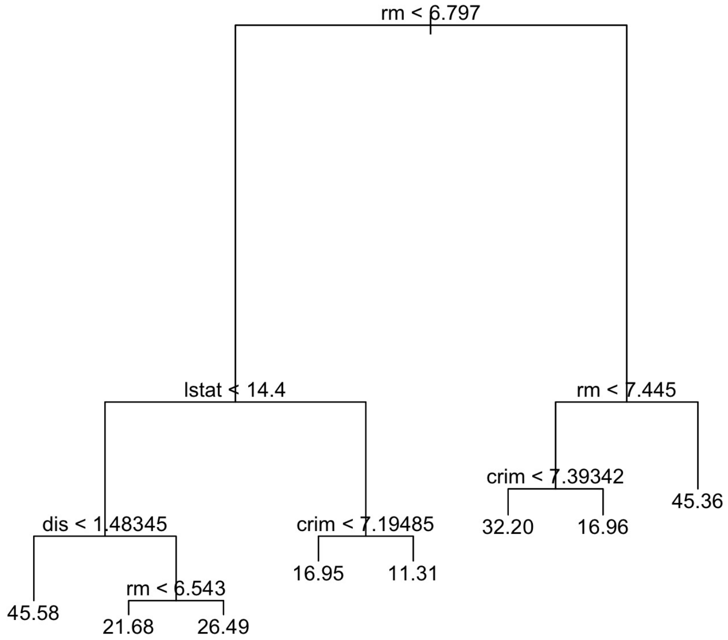
- For each value of α there corresponds a subtree $T \subset T0$ such that

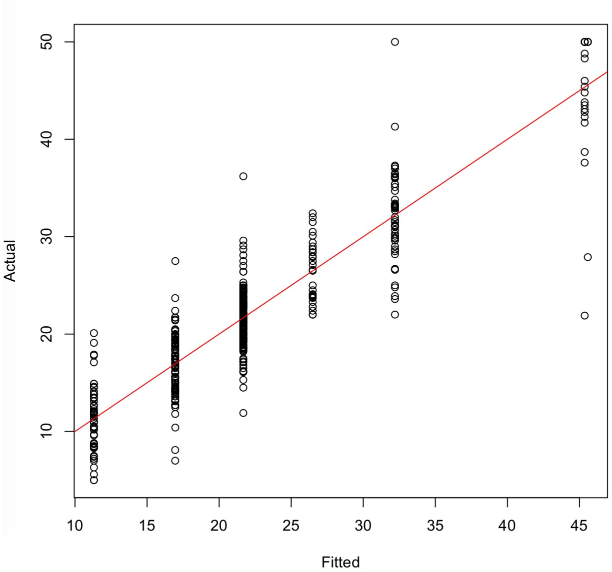
$$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} (y_i - \hat{y}_{Rm})^2 + \alpha|T|$$

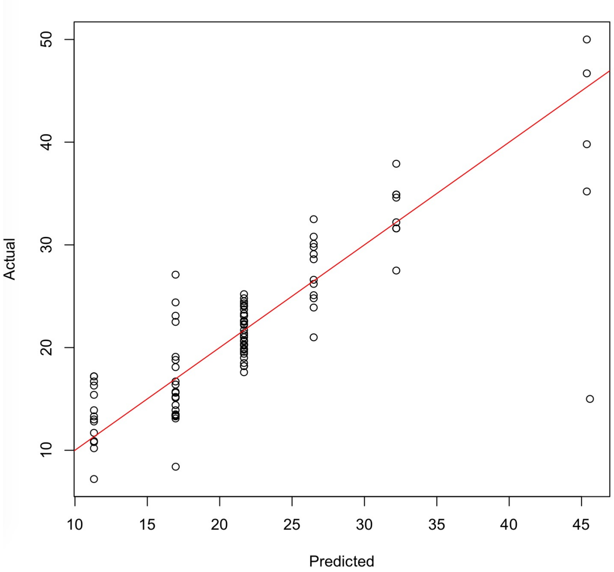
  is as small as possible.

- $|T|$ is the number of terminal nodes, Rm is the rectangle (i.e. the subset of predictor space) corresponding to the mth terminal node.

- Use CV to choose best α. (Algorithm 8.1 on p309)


rm < 6.797
lstat < 14.4
rm < 7.445
crim < 7.39342
45.36
dis < 1.48345
crim < 7.19485
32.20
16.96
16.95
11.31
rm < 6.543
45.58
21.68
26.49


Actual
Fitted
10
15
20
25
30
35
40
45
50

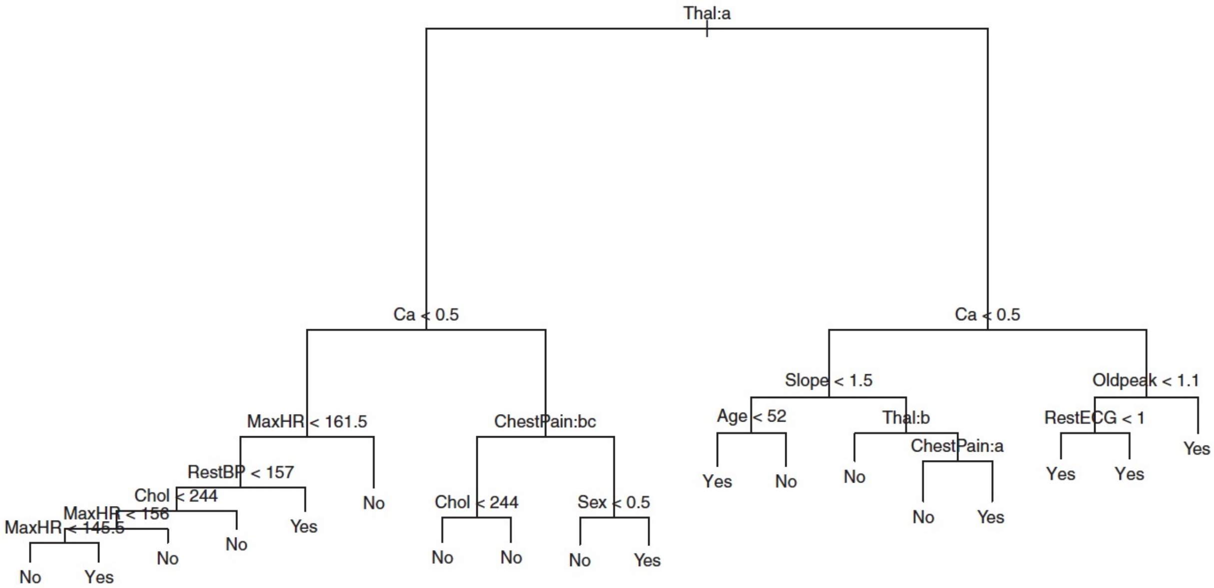

Actual
Predicted
10
15
20
25
30
35
40
45
50

Test.RSS

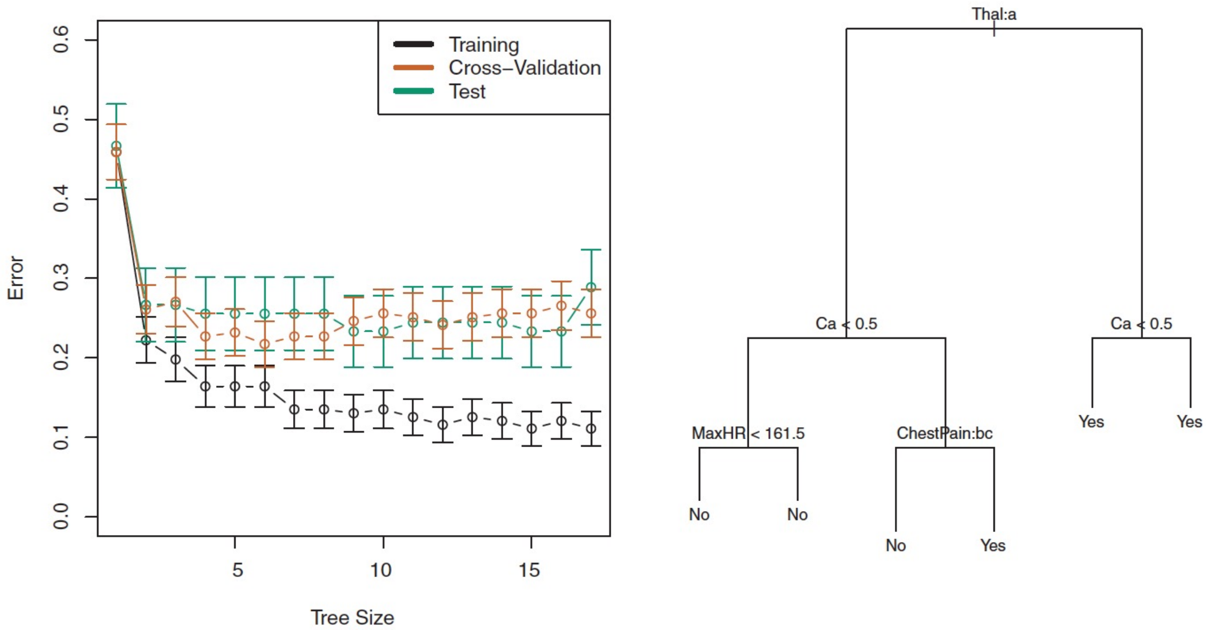
```
      RMSE   Rsquare
1 4.946511 0.6353647
```

A.7 Classification Trees

Heart data



Thal:a
Ca < 0.5
Ca < 0.5
MaxHR < 161.5
ChestPain:bc
Slope < 1.5
Oldpeak < 1.1
Age < 52
Thal:b
ChestPain:a
RestECG < 1
RestBP < 157
Chol < 244
MaxHR < 156
MaxHR < 145.5
Chol < 244
Sex < 0.5
No
Yes
No
No
Yes
No
No
No
No
Yes
Yes
No
No
No
Yes
Yes
Yes
Yes



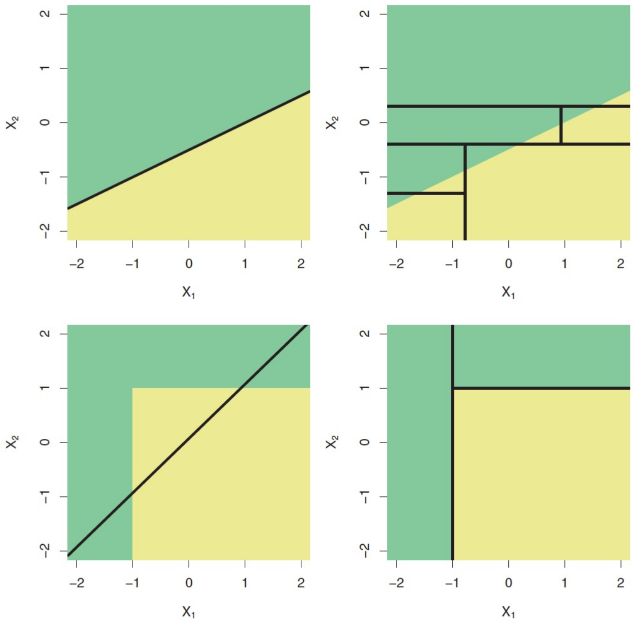
Training
Cross−Validation
Test
Error
Tree Size
0.0
0.1
0.2
0.3
0.4
0.5
0.6
5
10
15
Thal:a
Ca < 0.5
Ca < 0.5
MaxHR < 161.5
ChestPain:bc
Yes
Yes
No
No
No
Yes

A.8 Tree vs Linear Models

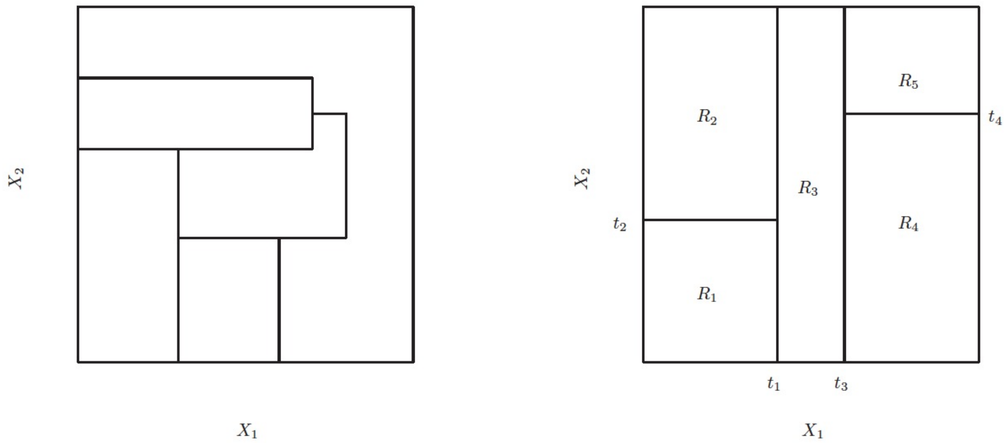
- $$f(X) = \beta_0 + \sum_{j=1}^{p} X_j \beta_j$$

  $$f(X) = \sum_{m=1}^{M} c_m \cdot I(x \in R_m)$$

- Trees are easy to interpret
- Trees may resemble how humans make decisions
- Trees can handle qualitative predictors easier than linear models
- However, trees in general does not have comparable predictive power as other models.



X_1
X_2
−2
−1
0
1
2

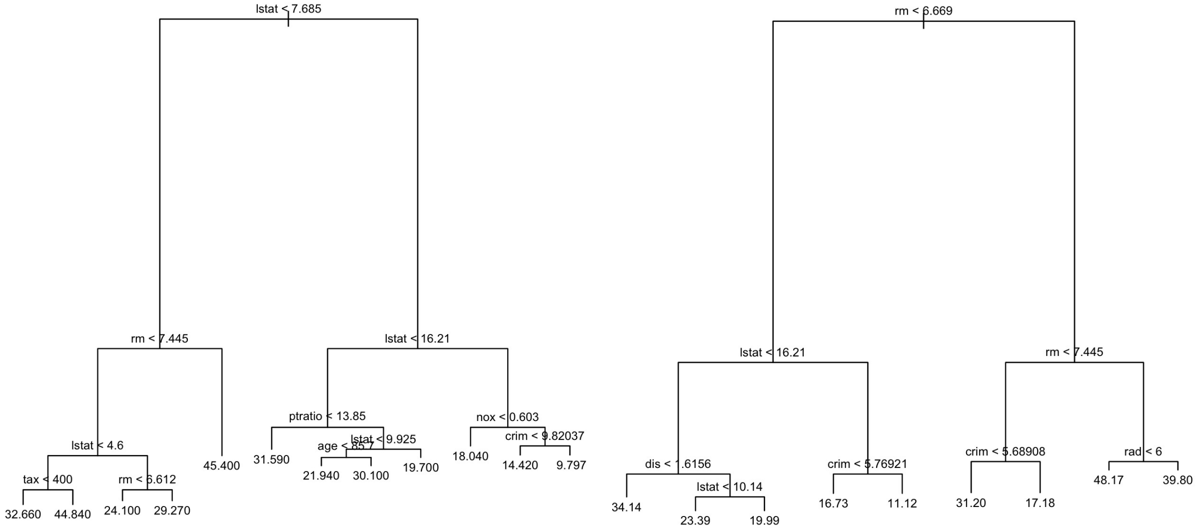

X_2
X_1
R_1
R_2
R_3
R_4
R_5
t_1
t_2
t_3
t_4
X_2
X_1

A.9 Bagging

- Trees have too large of variance
- Bagging = Bootstrap aggregation
- can be used outside of DT, but drastically improves DT.
- Use majority class rule for classification prediction
- may cause DT to lose the interpretability (no more Tree diagram)
- Record RSS decrease due to a single split, average over all bootstrapped samples.
- Out-of-Bag Error Estimation (Estimate Test error of Bagged model)
- When the training set for the current tree is drawn by sampling with replacement, about one-third of the cases are left out of the sample. $(e^{-1} = \lim(1 - 1/n)^n)$

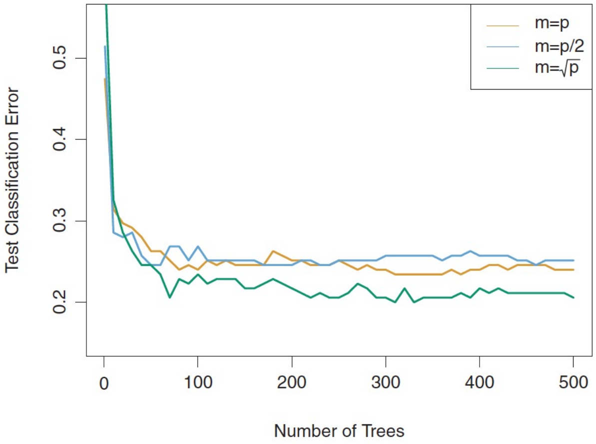
A.10 Large Variance

From Boston data CV1 and CV2


lstat < 7.685
rm < 7.445
lstat < 16.21
lstat < 4.6
tax < 400
rm < 6.612
32.660
44.840
24.100
29.270
45.400
ptratio < 13.85
31.590
lstat < 9.925
age < 85.7
21.940
30.100
19.700
nox < 0.603
18.040
crim < 9.82037
14.420
9.797
rm < 6.669
lstat < 16.21
dis < 1.6156
34.14
lstat < 10.14
23.39
19.99
crim < 5.76921
16.73
11.12
rm < 7.445
crim < 5.68908
31.20
17.18
rad < 6
48.17
39.80

A.11 Random Forests

- Like Bagging, but de-correlates
- Each time split occurs, only random sample of m predictor can be considered. Choose $m = \sqrt{p}$.



m=p
m=p/2
m=√p
Test Classification Error
0.2
0.3
0.4
0.5
0
100
200
300
400
500
Number of Trees

A.12 Boosting

- Set $\hat{f}(x) = 0$ and $r_i = y_i$ for all i in the training set.
- for $b = 1, 2, \ldots, B$ repeat:
  1. Fit a tree $\hat{f}^b$ with d splits (d=depth) ($d+1$ terminal nodes) to the training data (X, r).
  2. Update $\hat{f}$ by adding in a shrunken version of the new tree:

$$\hat{(}x) \leftarrow \hat{f}(x) + \lambda \hat{f}^b(x)$$

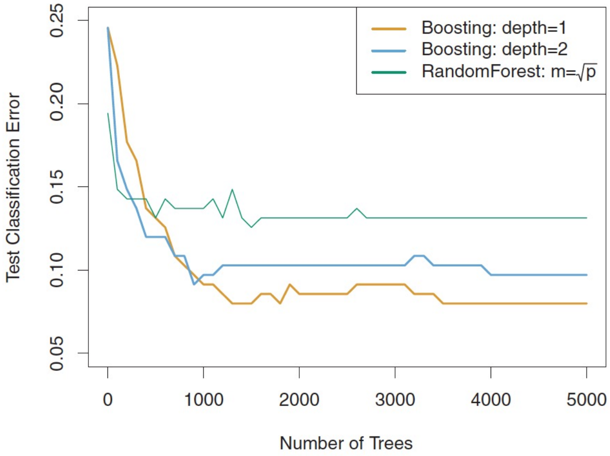
  3. Updata the residuals,

$$r_i \leftarrow r_i - \lambda \hat{f}^b(x_i)$$

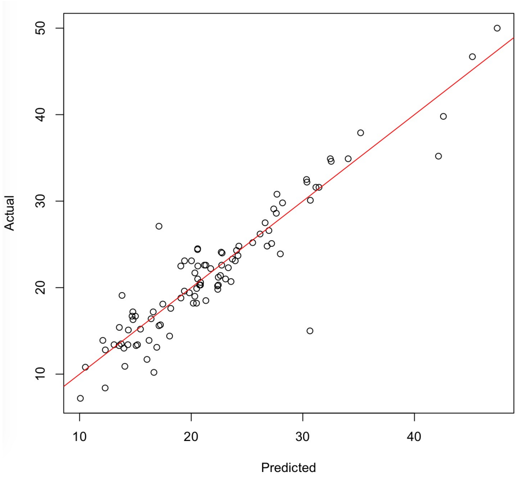
- Output the boosted model

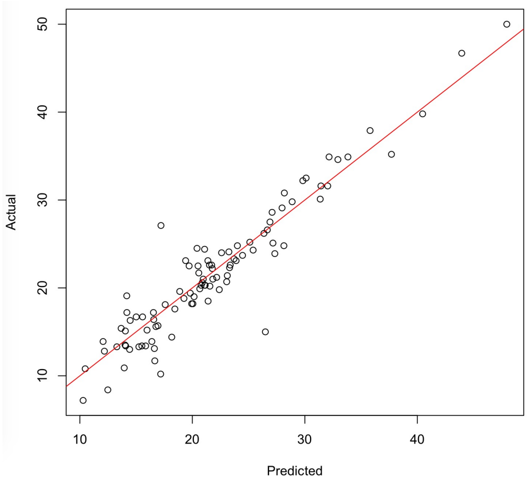
$$\hat{f}(x) = \sum_{b=1}^{B} \lambda \hat{f}^b(x)$$

- Boosting Learns SLOWLY.



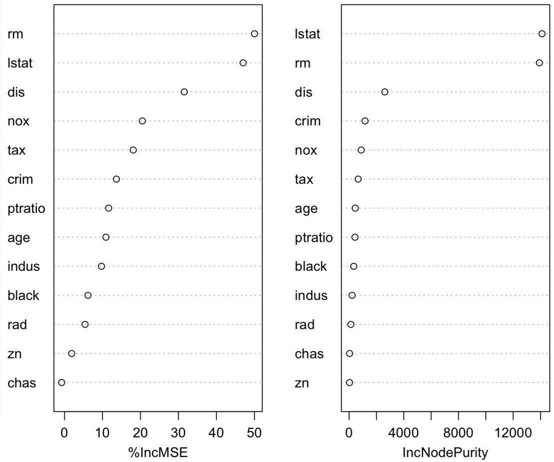
A.13 Boston Data





```
Bagging (m=13)
Test.RSS
         RMSE   Rsquare
medv 2.891555 0.8635642
```

```
Random Forest (m=6)
Test.RSS
         RMSE   Rsquare
medv 2.569209 0.8931857
```


rm
lstat
dis
nox
tax
crim
ptratio
age
indus
black
rad
zn
chas
0
10
20
30
40
50
%IncMSE
lstat
rm
dis
crim
nox
tax
age
ptratio
black
indus
rad
chas
zn
0
4000
8000
12000
IncNodePurity

A.14 party Package

```
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https://ademos.people.uic.edu/Chapter24.html

R package 'party::ctree()'

Uses ID3 Algorithm (Iterative Dichotomiser 3) created by JR Quinlan

  wiki: https://en.wikipedia.org/wiki/ID3_algorithm
    (note Examples -> Observations)

  Uses criteria of min entropy to grow trees. Does not prune.

  Some source say it looks at Information Gain,
  but max(IG) iff min(entropy), some other say.
```

IG formula is on Wiki.

Stopping criteria is one of following 3:

- every element in the subset belongs to the same class;
- there are no more attributes to be selected
- there are no examples in the subset,
  which happens when no example in the parent set was found to match a specific value of the selected attribute.
  An example could be the absence of a person among the population with age over 100 years.