650 Homework Assignment 3 - Poisson Processes

Due Thursday, July 14th

1. Suppose you have two meetings scheduled today, one 10 am and the other at 11 am. The amounts of time that the meeting takes are independent exponential random variables with mean of 1 hour.
   (a) Find probability that you will be late to the second meeting.
   (b) The second meeting does not start until you arrive. Find expected amount of time the second meeting must wait for you (wait is 0 if your 1st meeting is over before 11).
2. Suppose in a system, component failure occur according to a Poisson process with rate 1.5 per year, and the system fails when 20 such component failure have occurred. Find mean and variance of the system lifetime.
3. Suppose events occur according to a Poisson process with rate $\lambda = 2$ per hour.
   (a) What is the probability that no event occurs between 8 P.M. and 9 P.M. ?
   (b) Starting at noon, what is the expected time at which the 3rd event occurs?
   (c) What is the probability that three or more events occur between 1 P.M. and 4 P.M. ?
4. Teams 1 and 2 are playing a match. The teams score points according to independent Poisson processes with respective rates 3and 2.5. The match ends when one of the teams has scored 10 points. Find the probability that team 1 wins.
5. Customers can be served by any of three servers, where the service times of server i are exponentially distributed with rate $\mu_1 = 1/5, \mu_2 = 1/8, \mu_3 = 1/12$. Whenever a server becomes free, the customer who has been waiting the longest begins service with that server.
   (a) If you arrive to find all three servers busy and no one waiting, find the expected time until you depart the system.
   (b) Find variance of time until you depart the system in above case.
   (c) If you arrive to find all three servers busy and one person waiting, find the expected time until you depart the system.

6. There are two types of claims that are made to an insurance company. Number of type 1 claims and type 2 claims follow independent Poisson processes with rates $\lambda_1 = 10$ and $\lambda_2 = 3$ respectively. The amounts of successive type 1 claims can be \$1000 or \$2000 with probability .6 and .4 respectively, whereas the amounts from type 2 claims are \$2000 and \$4000 with probability .7 and .3 respectively.

   (a) Suppose a claim of unkown amount has been made. What is the probability that it is type 1? How about type 2?

   (b) Bayes' formula states that if claim of \$2000 is made, then probability that the claim was type 1 can be written as

$$P\big(\text{Type 1}\big|\$2000\big) = \frac{P\big(\$2000\big|\text{Type 1}\big)P(\text{Type 1})}{P\big(\$2000\big|\text{Type 1}\big)P(\text{Type 1}) + P\big(\$2000\big|\text{Type 2}\big)P(\text{Type 2})}.$$

   Calculate this probability.