## 650 Homework Assignment 4 - Poisson Process and Simulation

Due Monday, July 25th

1. Suppose passengers arrive at train station according to a poisson process with rate of 100 per hour. Cost of each passenger's waiting time is $\$ 8$ per hour. If each train cost $\$ 1000$ to run, what is the most cost-effective frequency of the train? Use expected cost as optimization criteria. That is, each time the train runs, total amount of passenger waiting cost should be as close as possible to $\$ 1000$.
(e.g. If 100 people all waited 1 h each, then total waiting cost is $\$ 800$.)
2. Let $X$ be a r.v. with CDF

$$
F(x)= \begin{cases}0 & x<0 \\ x^{4} & 0 \leq x \leq 1 \\ 1 & 1<x\end{cases}
$$

$F(x)=X^{4}, \quad 0<x<1$.
Use inverse method to gererate 5000 i.i.d. realization from CDF $F(x)$. Plot histogram of $X_{1}, \ldots, X_{5000}$ and overlay pdf function of r.v. $X$.
3. Consider the integral

$$
V=\int_{0}^{1} \int_{0}^{1} \frac{1}{|u-v|^{1-2 d}} d u d v
$$

where $d=.4$.
(a) Evaluate the integral using Monte Carlo integration method. Use iteration of 10,000. Obtain $95 \%$ Confidence Interval for the value of the integral.
(b) Use Antithetic Variate method to reduce the length of Confidence Interval without increasing the overall number of iteration.
(c) (Bonus question) Numerically obtain the value of $V$ using method other than MC integration. (You can use any software)

